A finite element framework for numerical simulation of multiphase granular flow

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Outline

- Introduction
  - Eulerian-Eulerian Modelling
  - Kinetic Theory
- Example: Particle settling in pipes
- Validation: transient flow in fluidized beds

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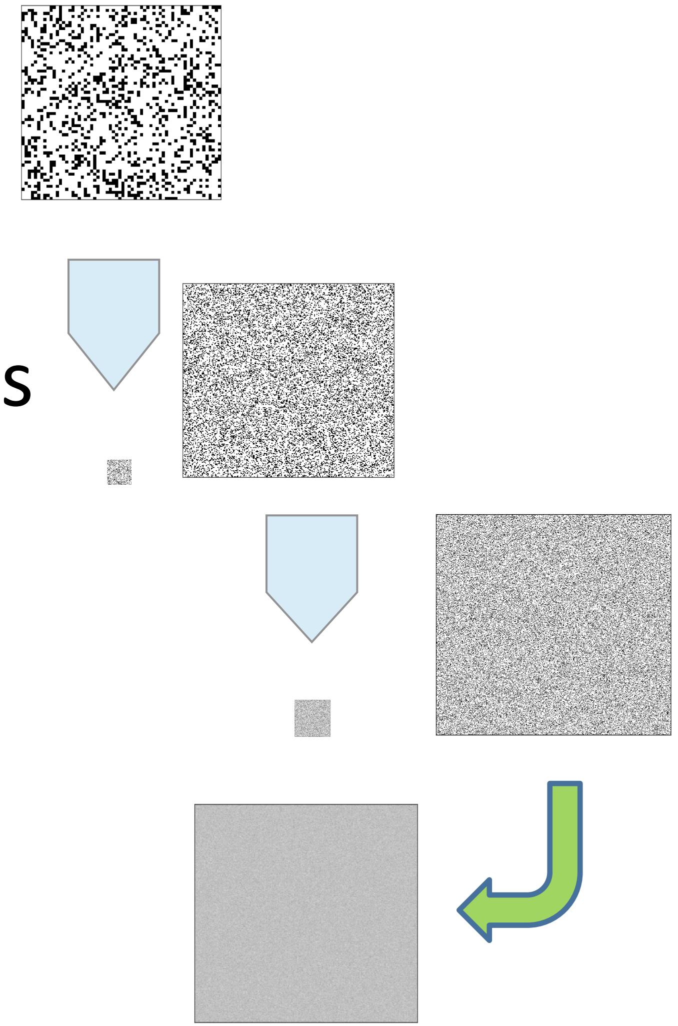
Eulerian-Eulerian modelling

Numerical model implies minimum length scale

Droplets/bubbles/solid particles may be far below this scale.

Average equations to homogenize into two-fluid model.



Eulerian-Eulerian modelling

General equations:

momentum:

$$\frac{\partial}{\partial t}(\rho_i \alpha_i \boldsymbol{u_i}) + \nabla \cdot (\rho_i \alpha_i \boldsymbol{u_i} \boldsymbol{u_i}) = -\alpha_i \nabla p_i - \rho_i \alpha_i g \hat{\boldsymbol{z}} + \boldsymbol{F_i}$$

continuity:

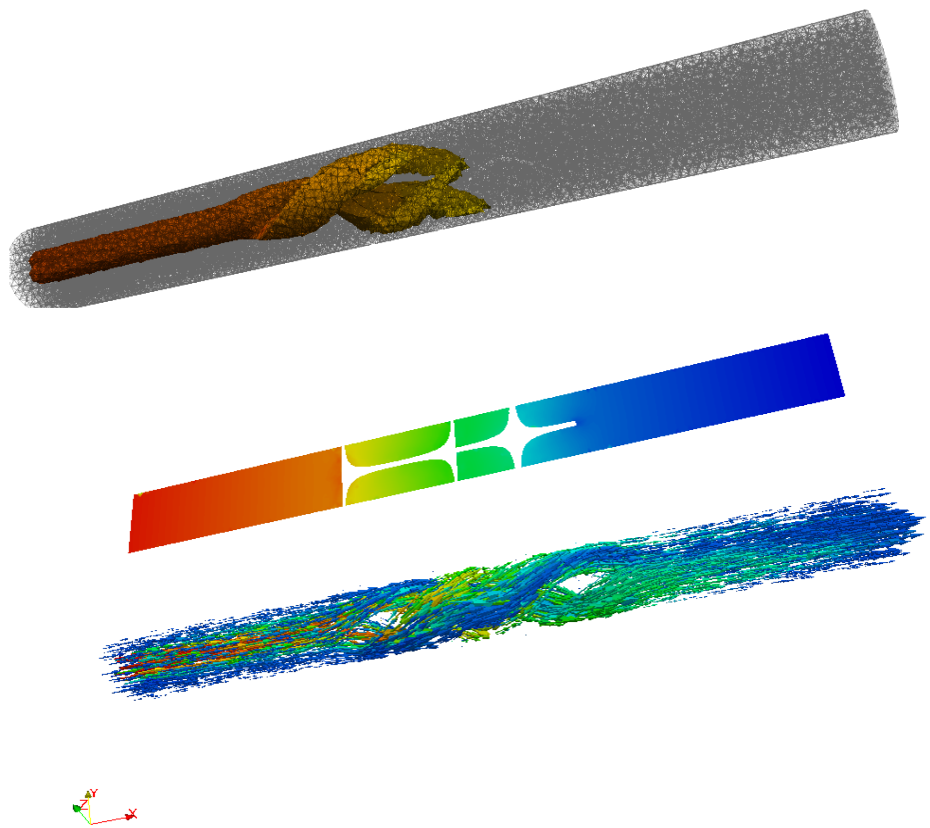
$$\frac{\partial}{\partial t}(\rho_i \alpha_i) + \nabla \cdot (\rho_i \alpha_i \boldsymbol{u_i}) = S_i$$

internal energy:

$$\frac{\partial}{\partial t}(\rho_i \alpha_i c_{p,i} T_i) + \nabla \cdot (\rho_i \alpha_i c_{p,i} T_i \boldsymbol{u_i}) + p_i \nabla \cdot \boldsymbol{\alpha_i u_i} = S_i$$

FLUIDITY

- Finite element solver framework
- Mesh adaptivity capability
- Multiphase & multimaterial formulations (and both together)

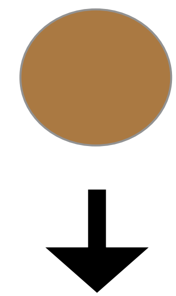


Interphase forces: drag

- Drag term: momentum exchange between phases

$$K_{\text{drag}} = \frac{\partial \boldsymbol{F}}{\partial \Delta \boldsymbol{u}} (\boldsymbol{u}_1 - \boldsymbol{u}_2)$$

- Limit of few, small particles: Stokes law
- Behaviour at higher concentrations less well determined.



$$v_{\text{terminal}} = \frac{2}{9} \frac{(\rho_d - \rho_c)}{\mu_c} g {d_p}^2$$

Interphase forces: drag

- Many drag closure models exist:
  - Schiller and Naumann (1935)
  - Ergun (1952)
  - Wen & Yu (1966)
  - Morsi and Alexander (1972)
  - Schwarz and Turner (1988)
  - Schuh et al. (1989)
  - Gidaspow(1990)

Interphase forces: drag

- Some more:
  - Richardson & Zaki (1954)
  - Symlaml & O’Brien (1987)
  - Hill Koch & Ladd (2006)
  - Du Plessis & Masliyah (1988)
- Variously from theory, numerical or empirical line matching with experiments

Interphase forces: drag

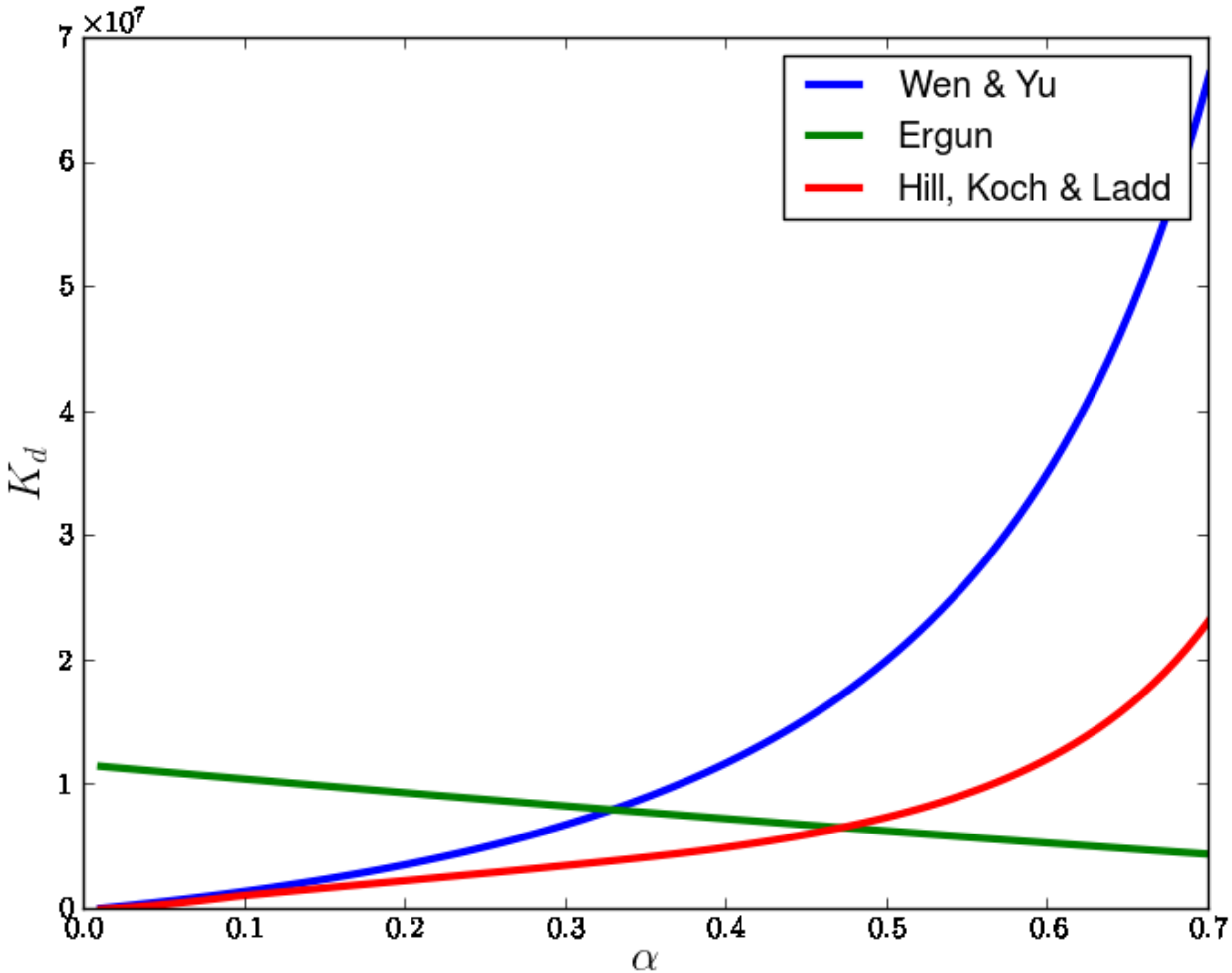
- Different closures appropriate for different systems
- Don't just implement a few, implement a framework

$$K_{\text{drag}} = \frac{\partial \boldsymbol{F}}{\partial \Delta \boldsymbol{u}} (\boldsymbol{u}_1 - \boldsymbol{u}_2)$$

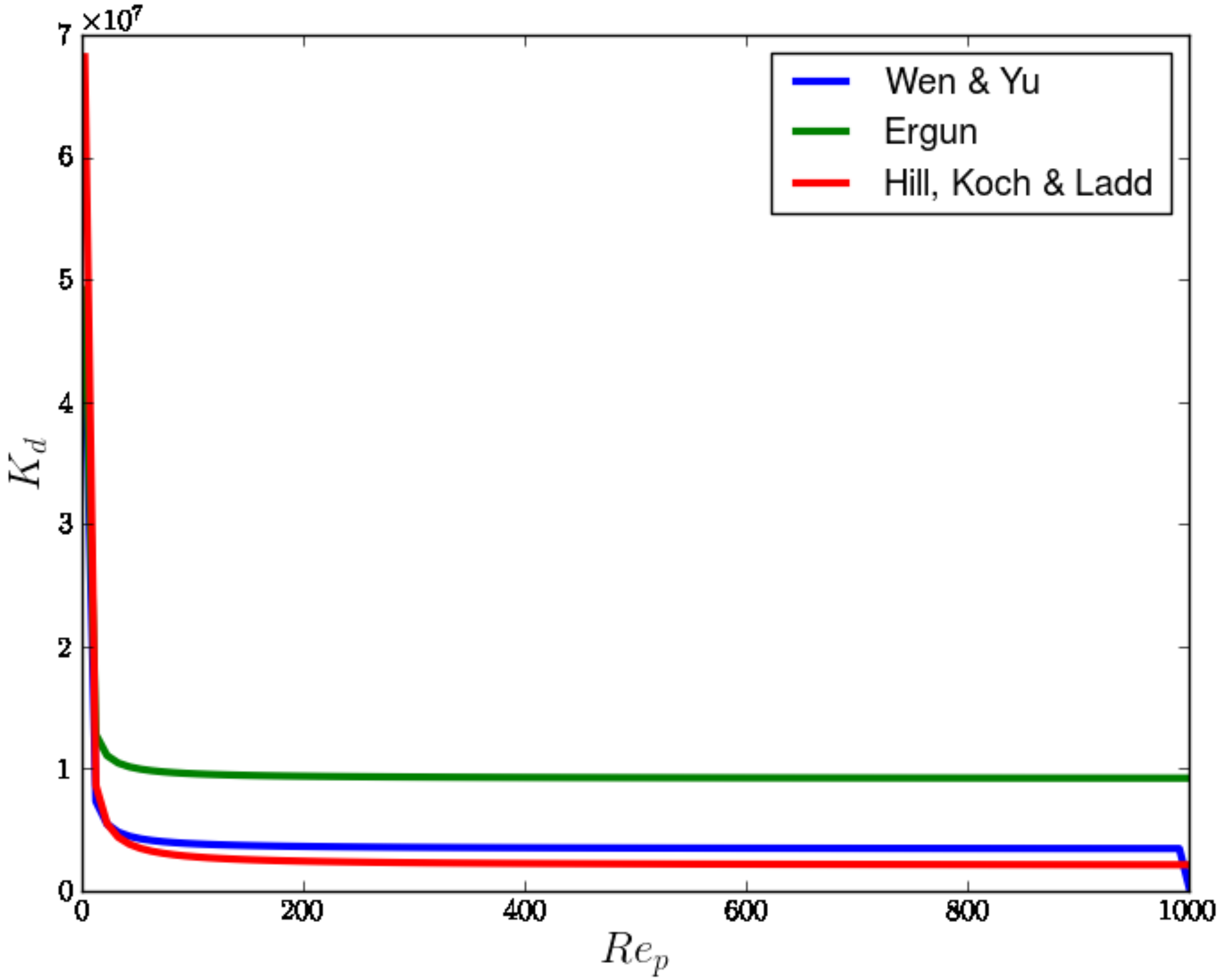
$$\frac{\partial \boldsymbol{F}}{\partial \Delta \boldsymbol{u}} = f(\alpha_i, \boldsymbol{u}_i, \rho_i)$$

- Specify through python code

Interphase forces: drag



Interphase forces: drag



Interphase forces: drag

Example: Wen & Yu drag correlation

```
def val(x,t,a_1,a_2,rho_1,rho_2,u_1,u_2):
   mu=1.0e-5
   d_s=150e-6
   Re_s=a_1*rho_1*abs(u_1-u_2)*d_s/mu
   if Re_s>1000:
       C_D=0.44
   else:
       C_D=24/Re_s*(1.0+0.15*(Re_s**0.687))
   return 3*C_D/4.0*a_1*a_2*rho_1*abs(u_1-u_2)/d_s*a_1**-2.65
```

Interphase forces: granular temperature

- Specialize to fluid-soild systems
- Solid has
  - Max packing density
  - Quasi-elastic collisions
  - Additional kinetic energy in fluctuations

λKinetic theory

λqv. Gidaspow(1994)

$$\boldsymbol{F}_{\text{solid}} = \nabla p_s + K_{\text{drag}} + \boldsymbol{F}_{\text{friction}}$$

$$p_s = \rho_s \alpha_s \left(1 + 2\left(1+e\right)\alpha_s g_0\right)\Theta_s$$

$$g_0 = \left(1 - \left(\frac{\alpha_s}{\alpha_{s,\max}}\right)^{\frac{1}{3}}\right)^{-1}$$

Interphase forces: granular temperature

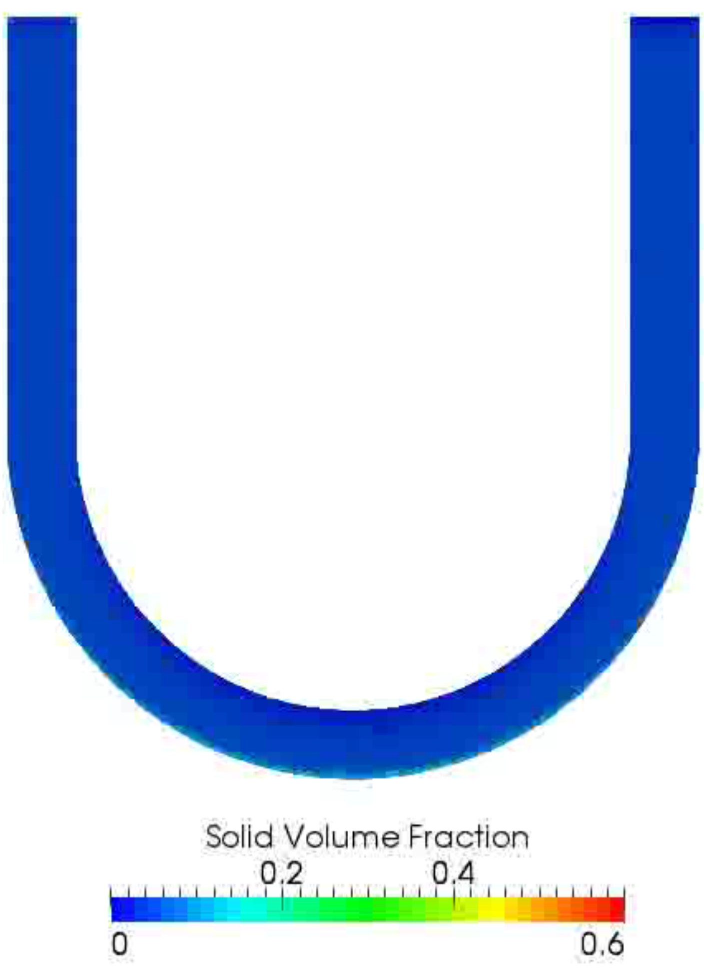
$$\boldsymbol{F}_{\text{friction}} = \alpha_s \nabla \cdot \left(\mu_s \left(\nabla \boldsymbol{u}_s + \nabla \boldsymbol{u}_s \right) + \left(\lambda_s - \frac{2\mu_s}{3} \nabla \cdot \boldsymbol{u}_s \right) \boldsymbol{I} \right)$$

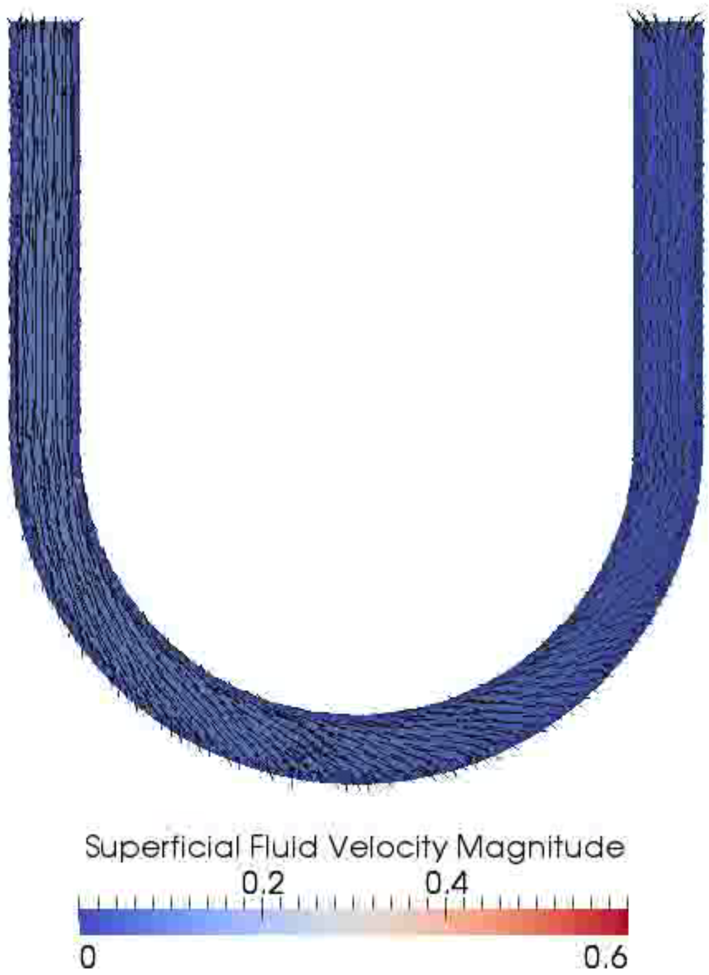
$$\mu_s = \frac{4}{5} \alpha_s d_s g_0 \left(1 + e\right) \left(\frac{\Theta_s}{\pi} \right)^{\frac{1}{2}}$$

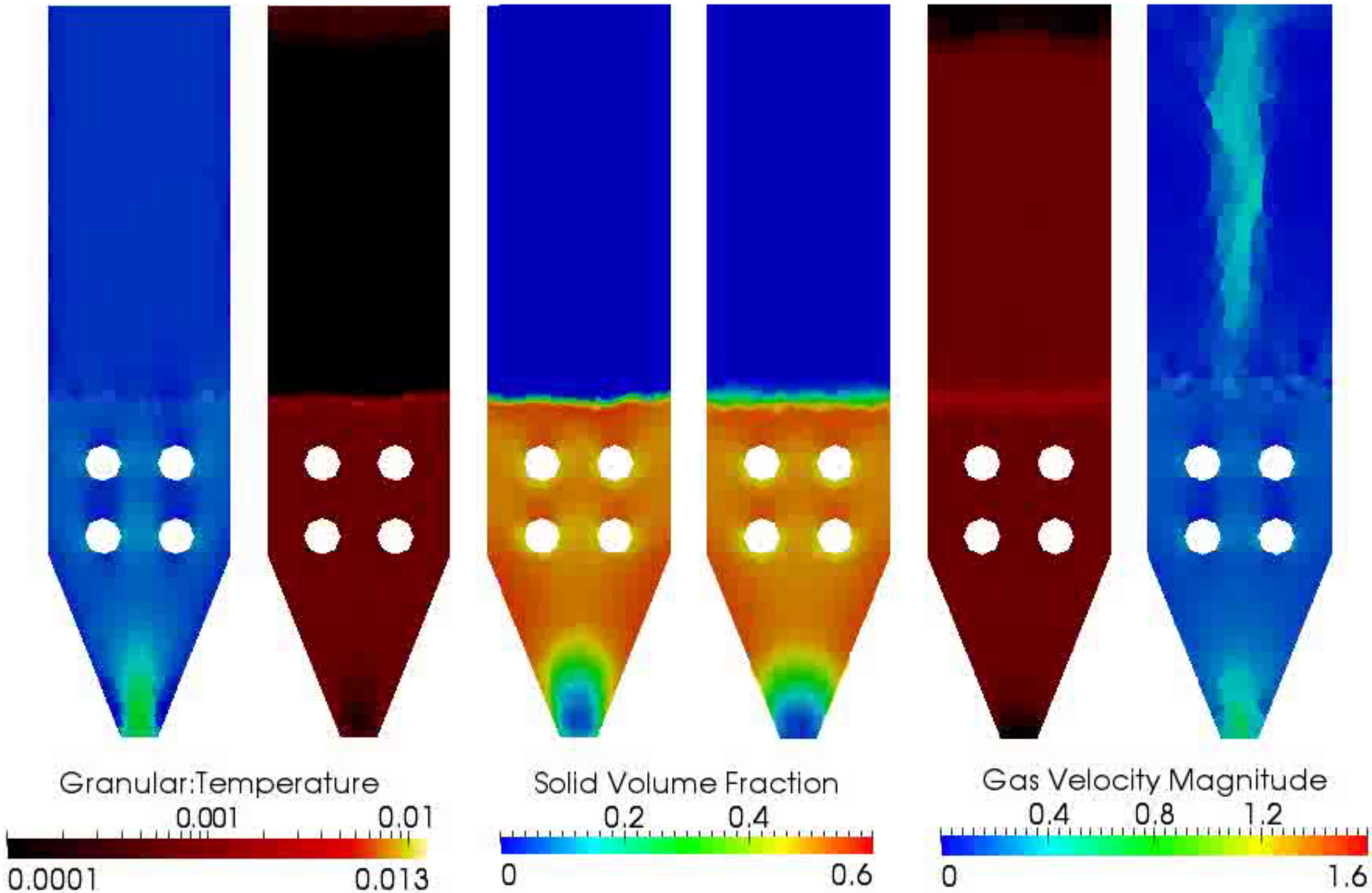
$$\lambda_s = \frac{4}{3} \alpha_s d_s g_0 \left(1 + e\right) \left(\frac{\Theta_s}{\pi} \right)^{\frac{1}{2}}$$

$$\frac{\partial}{\partial t} \left(\rho_s \alpha_s \Theta \right) + \nabla \cdot \left(\rho_s \alpha_s \Theta \boldsymbol{u}_s \right) = \nabla \boldsymbol{u}_s \cdot \tau_s - \gamma \Theta - \frac{\partial \boldsymbol{F}}{\partial \Delta \boldsymbol{u}} \Theta$$

Example: settling in pipes

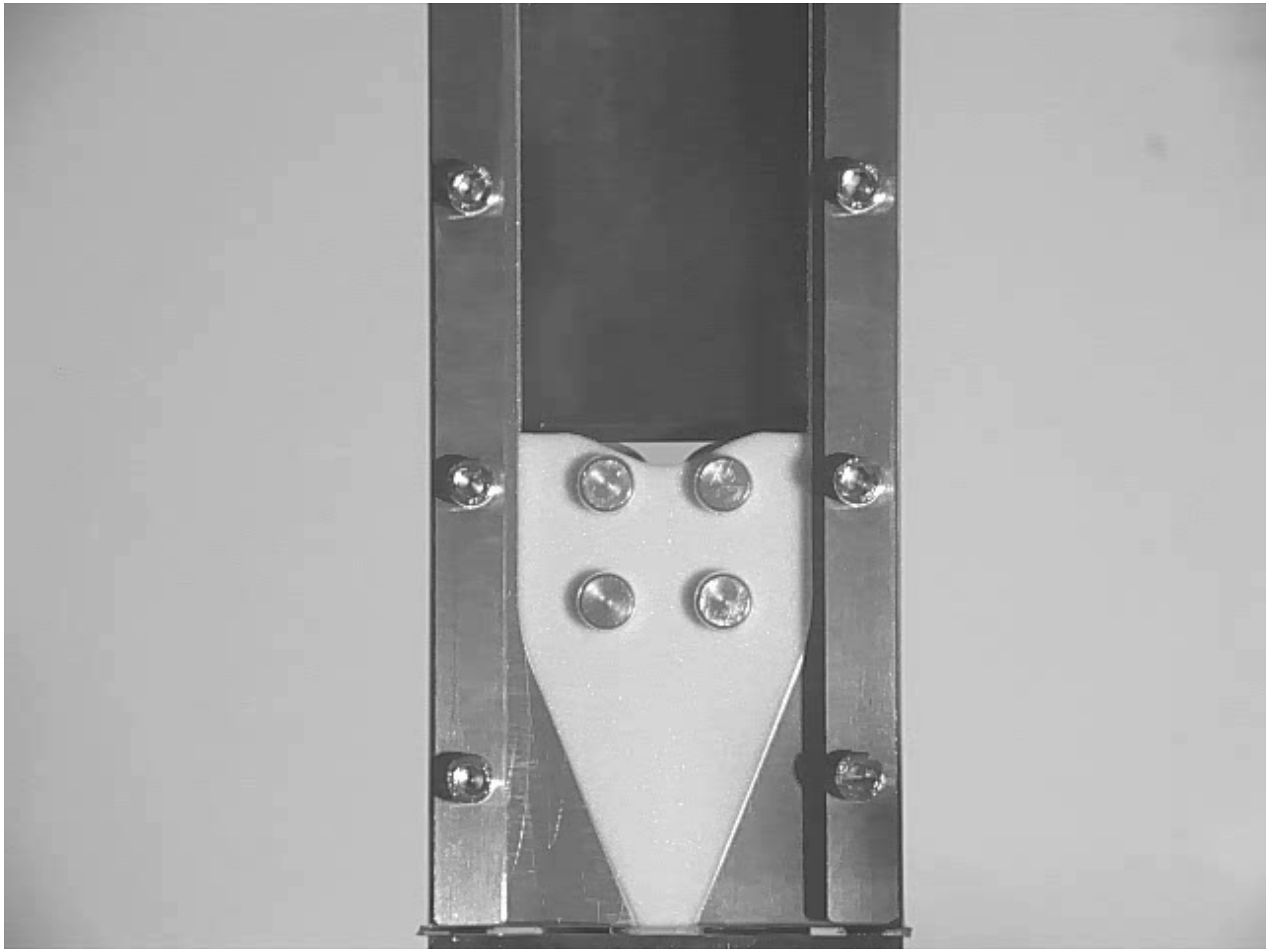





Granular:Temperature
0.001
0.01
0.0001
0.013
Solid Volume Fraction
0.2
0.4
0
0.6
Gas Velocity Magnitude
0.4
0.8
1.2
0
1.6

Example fluidized beds

Experimental reactor
M. Sakai
Univ Tokyo



Conclusions

- Demonstrated extensible CVFEM fluid-solid modelling framework with mesh adaptive capability
- applied to:
  - particle settling
  - granular flows
- Stepping stone to fluid-fluid model.

Future work

Future work:

- Polydispersion
- Deformable droplets
    - Adaptive Fluid-Fluid Modelling
- Rate equations
    - Chemistry
    - combustion
- Coupling with interfacial model.

Polydispersion

Multiple scales of dispersed phase material

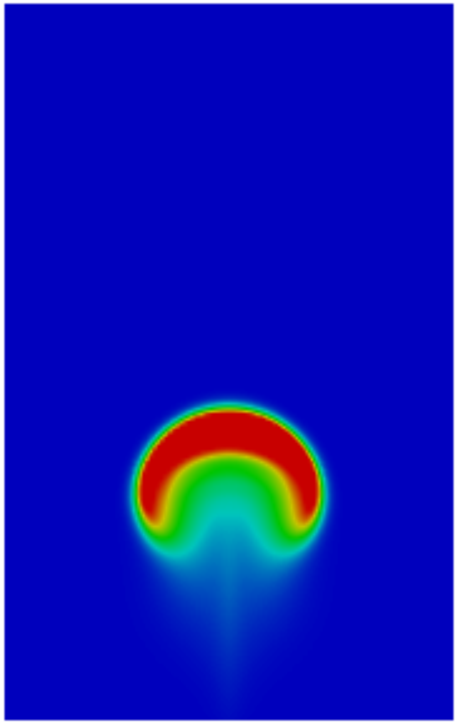
- Drag term:
  - technology already there!
- Particle particle interactions
- Size parameterization:
  - Binning approach
- Coupled phase & species models
  - (eg. MUSIG).

Deformable droplets

Closure models dependent on length parameter

$$\mathrm{Re}_s = \frac{\alpha_f \rho_f |u_f - u_s| d_s}{\mu_g}$$

Fluid droplets deform under action of interphase forces.



Rate equations

Couple dynamical core
to parameterized models
for mass /heat exchange

$$\frac{\partial}{\partial t}(\rho_i \alpha_i) + \nabla \cdot (\rho_i \alpha_i \boldsymbol{u_i}) = S_i$$

$$\frac{\partial}{\partial t}(\rho_i \alpha_i c_{p,i} T_i) + \nabla \cdot (\rho_i \alpha_i c_{p,i} T_i \boldsymbol{u_i}) + p_i \nabla \cdot \boldsymbol{\alpha_i u_i} = S_i$$

Coupling with interfacial model

May observe four phase problem

1. 1st continuous (eg. water)
2. 1st dispersed (eg. oil in water)
3. 2nd dispersed (water in oil)
4. 2nd continuous (oil)