



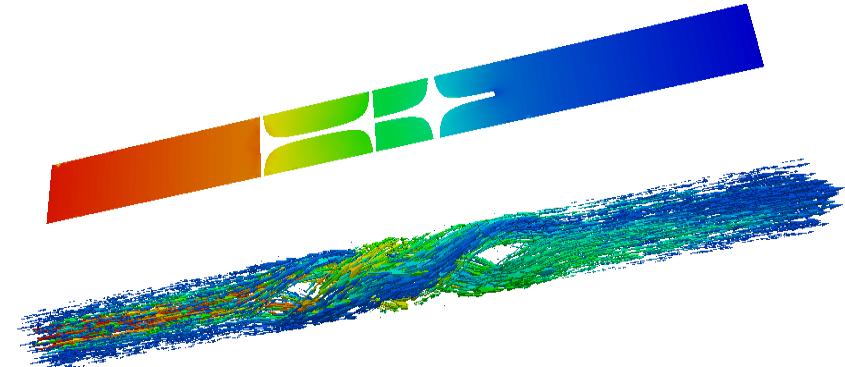
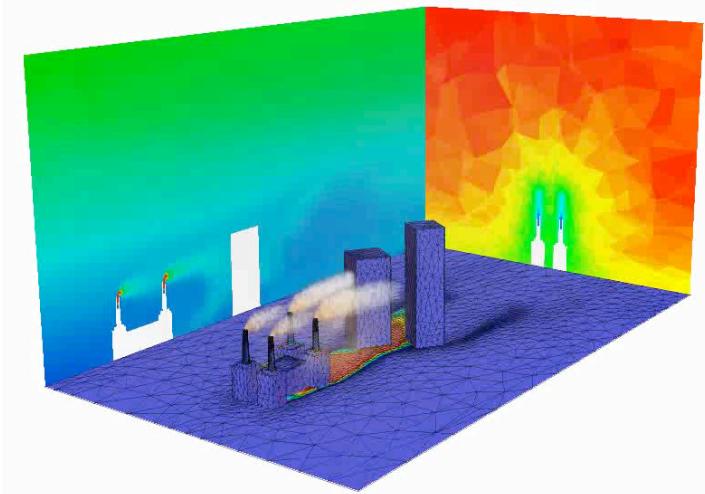
A novel finite element framework for numerical simulation of fluidization processes & multiphase granular flow

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Jefferson Gomes, Chris Pain, Omar Matar
Imperial College London
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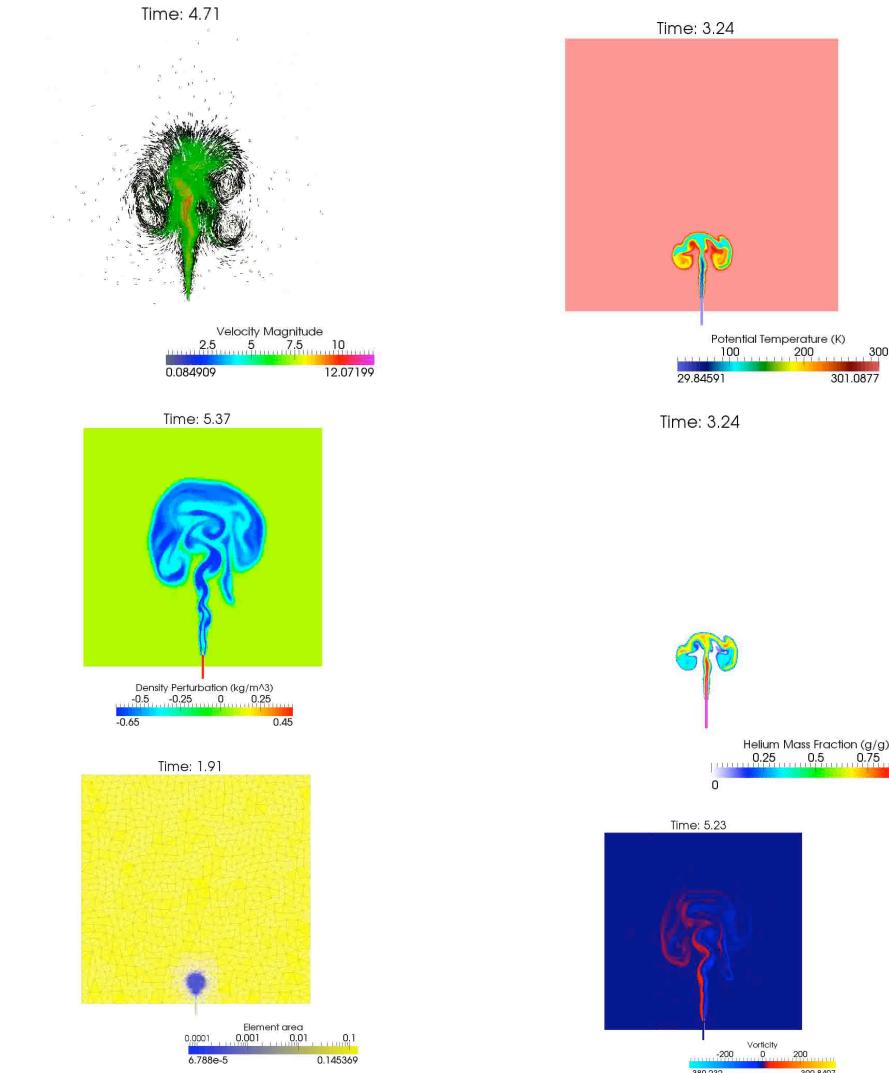
FLUIDITY

- (CV)FEM based Navier Stokes/Darcy flow solver framework
- Mesh adaptivity capability
- Multimaterial & multiphase formulations (and both together)



Mesh adaptivity

- Hessian approach optimizes estimate of linear interpolation error in chosen variables for given number of DOFs

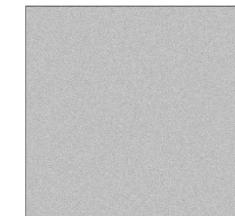
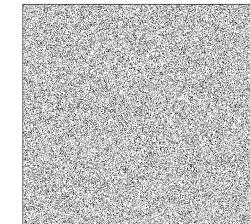
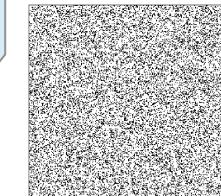


Eulerian-Eulerian modeling

Numerical modeling implies
existence minimum resolvable
length scale

Droplets/bubbles/solid particles
may be far below this scale.

Filter equations to homogenize
into “two-fluid” model.



Eulerian-Eulerian modeling

General equations:

momentum:

$$\frac{\partial}{\partial t} (\rho_i \alpha_i \mathbf{u}_i) + \nabla \cdot (\rho_i \alpha_i \mathbf{u}_i \mathbf{u}_i) = -\alpha_i \nabla p_i - \rho_i \alpha_i g \hat{\mathbf{z}} + \mathbf{F}_i$$

continuity:

$$\frac{\partial}{\partial t} (\rho_i \alpha_i) + \nabla \cdot (\rho_i \alpha_i \mathbf{u}_i) = S_i$$

internal energy:

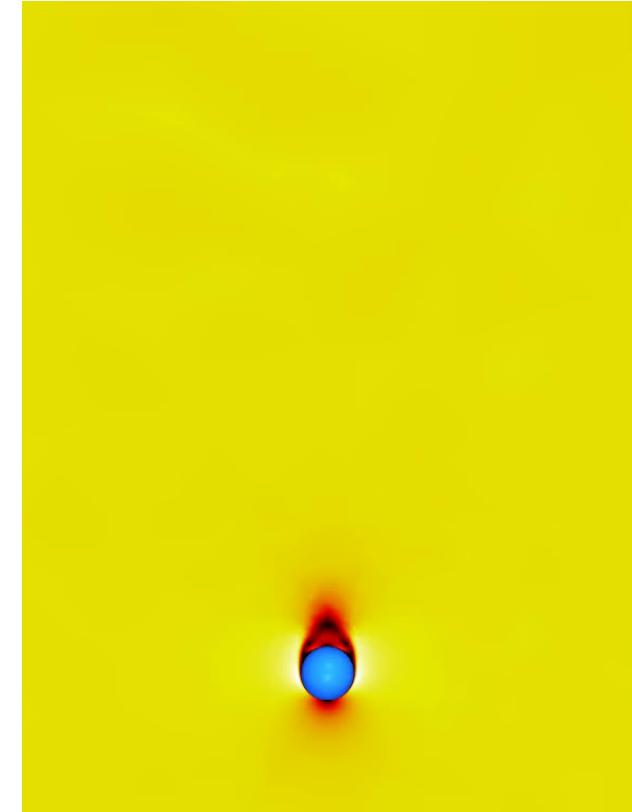
$$\frac{\partial}{\partial t} (\rho_i \alpha_i c_{p,i} T_i) + \nabla \cdot (\rho_i \alpha_i c_{p,i} T_i \mathbf{u}_i) + p_i \nabla \cdot \alpha_i \mathbf{u}_i = S_i$$

Interphase forces: drag

- Drag term: momentum exchange between phases

$$K_{\text{drag}} = \frac{\partial \mathbf{F}}{\partial \Delta \mathbf{u}} (\mathbf{u}_1 - \mathbf{u}_2)$$

- In limit of few, small particles: Stokes law
- Behaviour at higher concentrations less well determined.



$$v_{\text{terminal}} = \frac{2}{9} \frac{(\rho_d - \rho_c)}{\mu_c} g d_p^2$$

Interphase forces: drag

Many drag closure models exist:

- Schiller and Naumann (1935)
- Ergun (1952)
- Wen & Yu (1966)
- Symlaml & O'Brien (1987)
- Gidaspow(1990)
- Hill Koch & Ladd (2006)

Variously from theory, numerical
modelling or matching experiments.

Interphase forces: drag

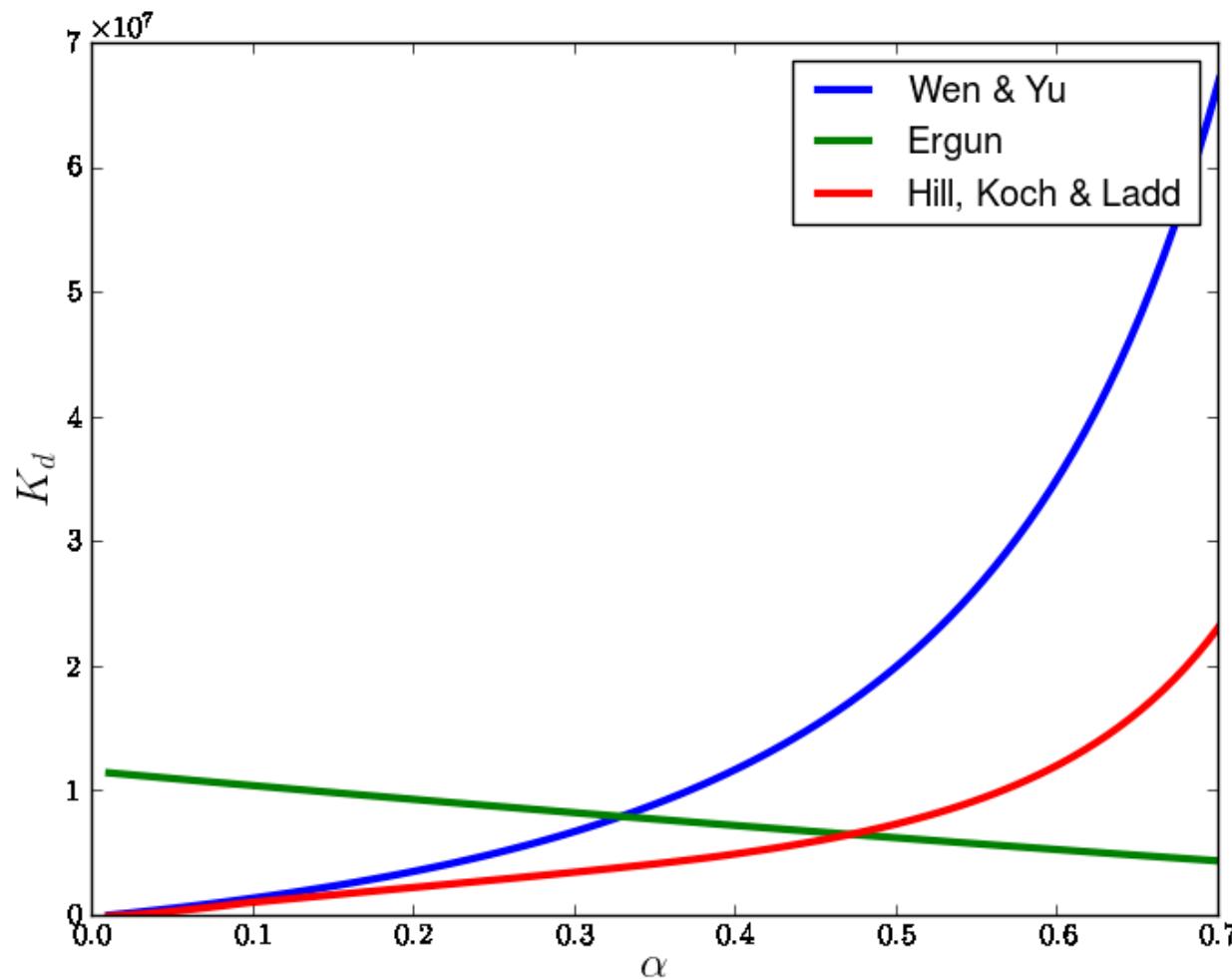
- Different closures appropriate for different systems
- Don't just implement a few, implement a framework

$$K_{\text{drag}} = \frac{\partial \mathbf{F}}{\partial \Delta \mathbf{u}} (\mathbf{u}_1 - \mathbf{u}_2)$$

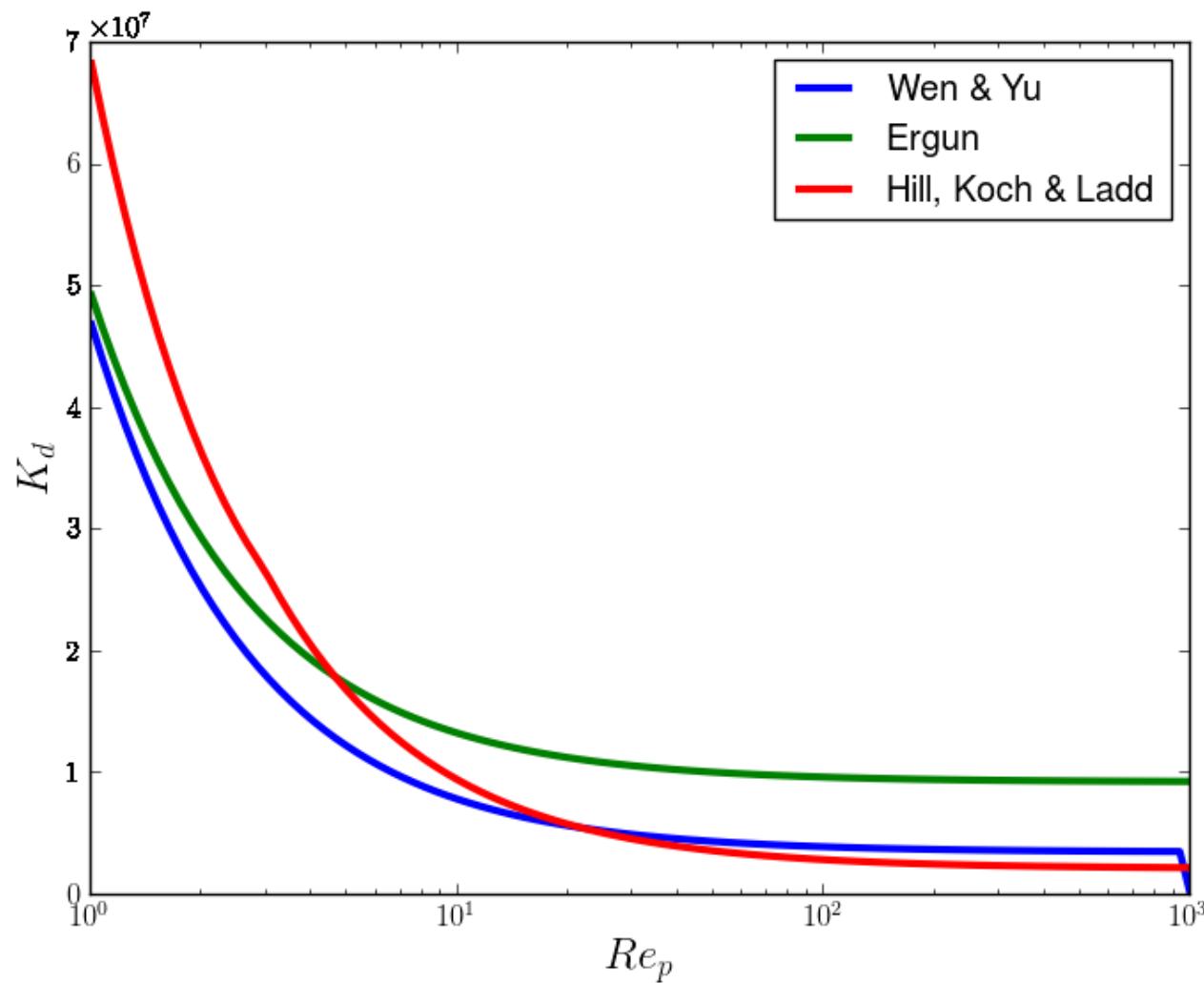
$$\frac{\partial \mathbf{F}}{\partial \Delta \mathbf{u}} = f(\alpha_i, \mathbf{u}_i, \rho_i)$$

- Specify through scripting code

Interphase forces: drag



Interphase forces: drag



Interphase forces: drag

Example: Wen & Yu drag correlation

```
def val(x,t,a_1,a_2,rho_1,rho_2,u_1,u_2):
    mu=1.0e-5
    d_s=150e-6
    Re_s=a_1*rho_1*abs(u_1-u_2)*d_s/mu
    if Re_s>1000:
        C_D=0.44
    else:
        C_D=24/Re_s*(1.0+0.15*(Re_s**0.687))
    return 3*C_D/4.0*a_1*a_2*rho_1*abs(u_1-u_2)/d_s*a_1**-2.65
```

Interphase forces: granular temperature

- Specialize to dense fluid-solid systems
- Solid has
 - Max packing density
 - Quasi-elastic collisions
 - Additional kinetic energy in fluctuations
- Kinetic theory closure
- qv. Gidaspow(1994)

$$\mathbf{F}_{\text{solid}} = \nabla p_s + K_{\text{drag}} + \mathbf{F}_{\text{friction}}$$

$$\Theta_s = \frac{1}{3} \langle |\mathbf{u} - \mathbf{v}|^2 \rangle$$

$$p_s = \rho_s \alpha_s (1 + 2(1+e) \alpha_s g_0) \Theta_s$$

$$g_0 = \left(1 - \left(\frac{\alpha_s}{\alpha_{s,\max}} \right)^{\frac{1}{3}} \right)^{-1}$$

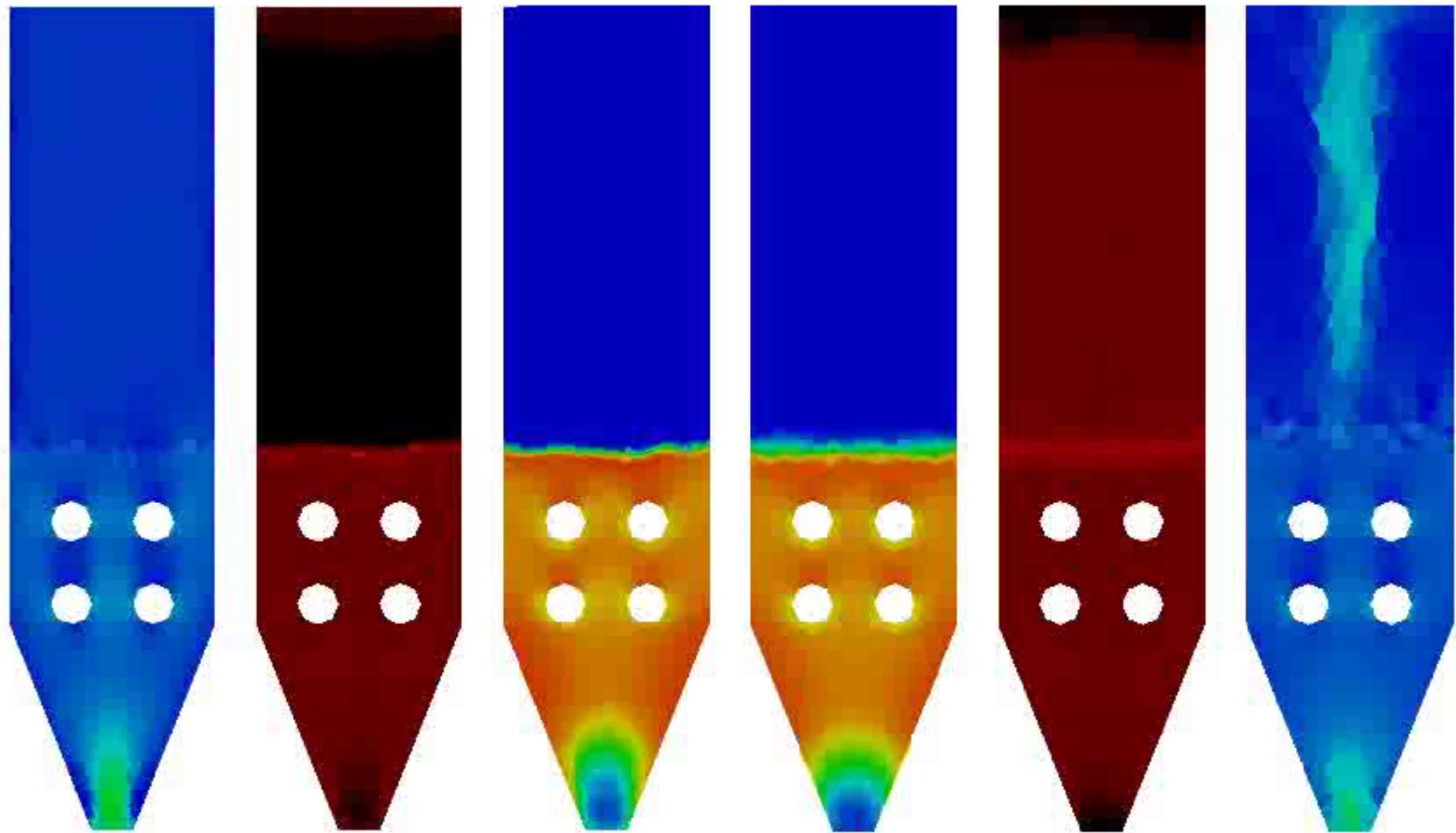
Interphase forces: granular temperature

$$\mathbf{F}_{\text{friction}} = \alpha_s \nabla \cdot \left(\mu_s (\nabla \mathbf{u}_s + \nabla \mathbf{u}_s) + \left(\lambda_s - \frac{2\mu_s}{3} \nabla \cdot \mathbf{u}_s \right) \mathbf{I} \right)$$

$$\mu_s = \frac{4}{5} \alpha_s d_s g_0 (1 + e) \left(\frac{\Theta_s}{\pi} \right)^{\frac{1}{2}}$$

$$\lambda_s = \frac{4}{3} \alpha_s d_s g_0 (1 + e) \left(\frac{\Theta_s}{\pi} \right)^{\frac{1}{2}}$$

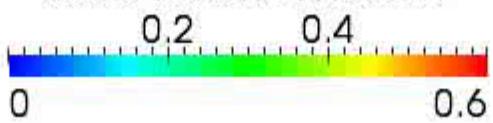
$$\frac{\partial}{\partial t} (\rho_s \alpha_s \Theta) + \nabla \cdot (\rho_s \alpha_s \Theta \mathbf{u}_s) = \nabla \mathbf{u}_s \cdot \boldsymbol{\tau}_s - \gamma \Theta - \frac{\partial \mathbf{F}}{\partial \Delta \mathbf{u}} \Theta$$



Granular Temperature



Solid Volume Fraction

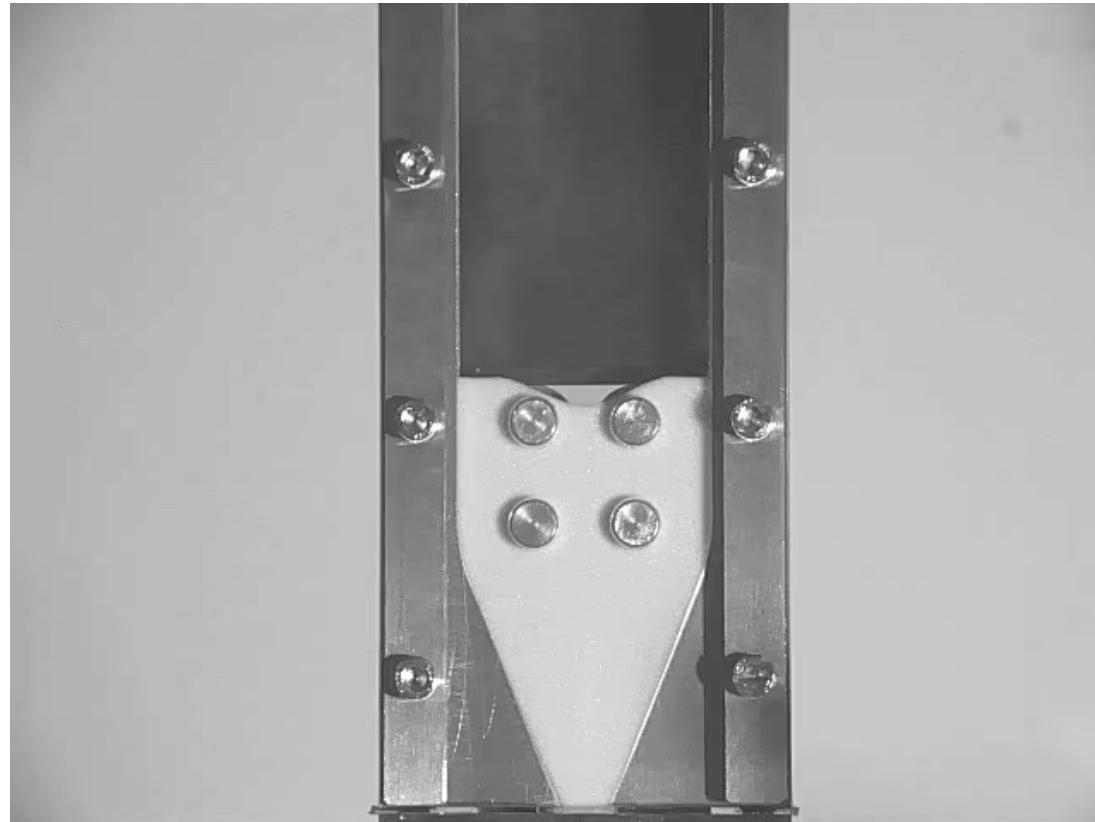


Gas Velocity Magnitude



Fluidized bed verification

Experimental
reactor in Lab
Mikio Sakai
Univ. Tokyo



Polydispersion

Multiple sizes/shapes of dispersed material

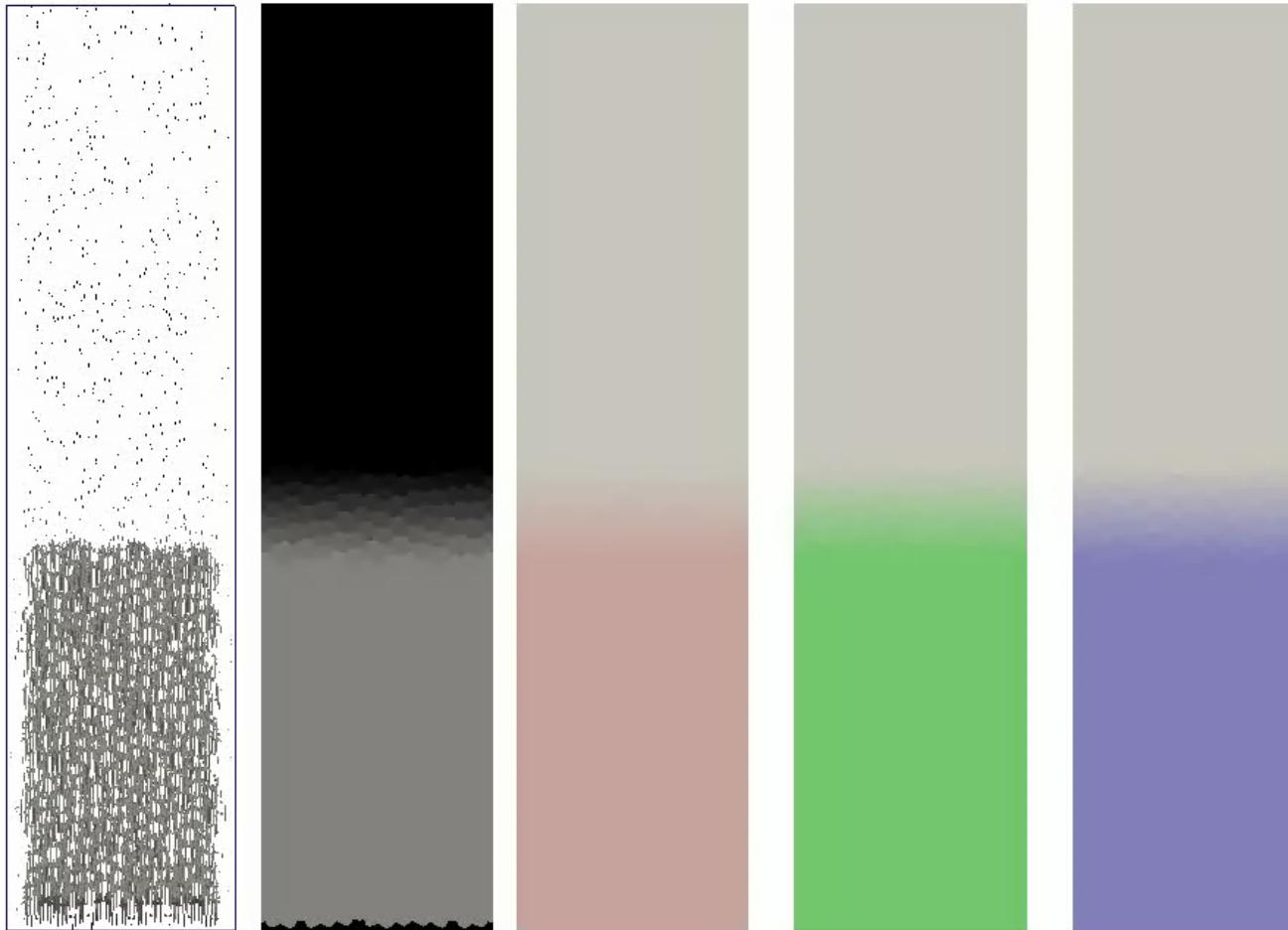
- Drag terms:
 - technology already there!
- Particle-particle interactions
- Size parameterization:
 - Binning approach

Polydispersion

Little consensus in literature:

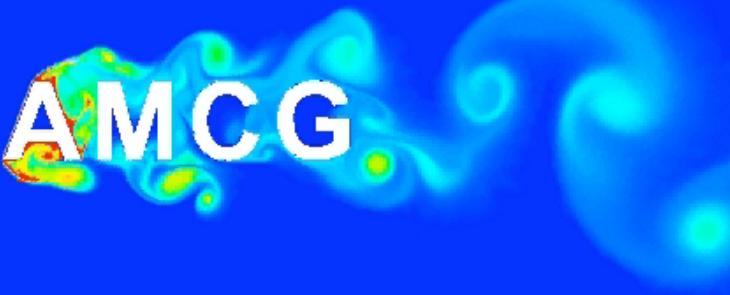
- Single or multiple granular temperatures?
- Modification of maximum packing fraction for mixtures?
- Existing gas-solid drag term, or additional terms?
- Which solid-solid interactions to consider?
 - Solid-solid interphase drag
 - Which phases contribute to solid viscosity?

Polydispersed Granular Mixtures: Single temperature



Future work

- Extensions to non-spherical and to deformable media
- Coupling with source/rate models
 - Chemistry
 - Combustion
- Coupled phase & species models



Thank you for your attention
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