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memphis multiphase

AMCG

A mesh-adaptive, Eulerian-Eulerian fluidized bed model with balanced finite element control volume methods applied to analyze particle size scaling laws

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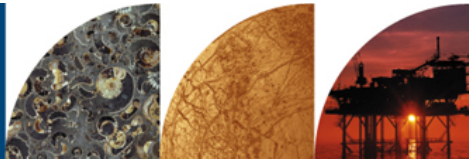
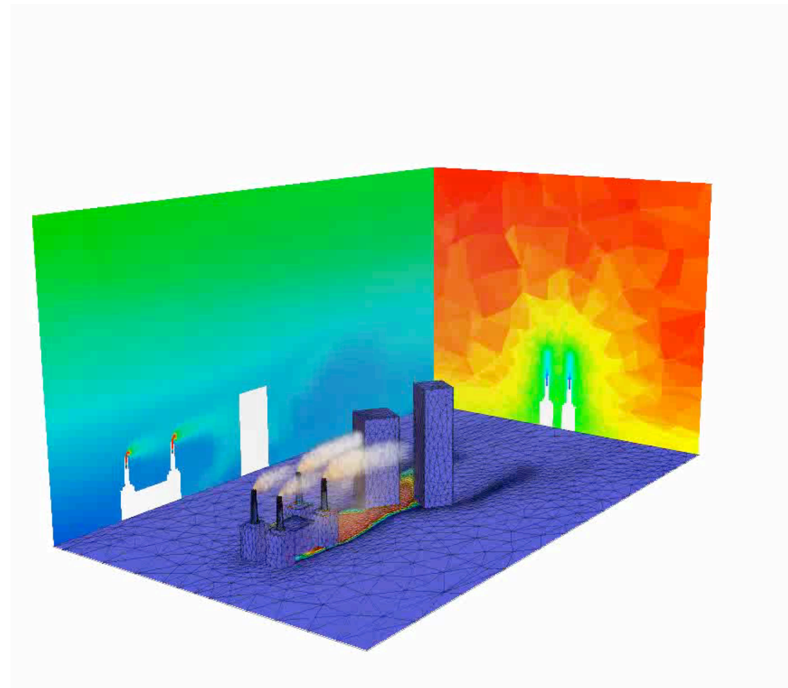
APCOM/ISCM 2013, Wednesday 11th December 2013

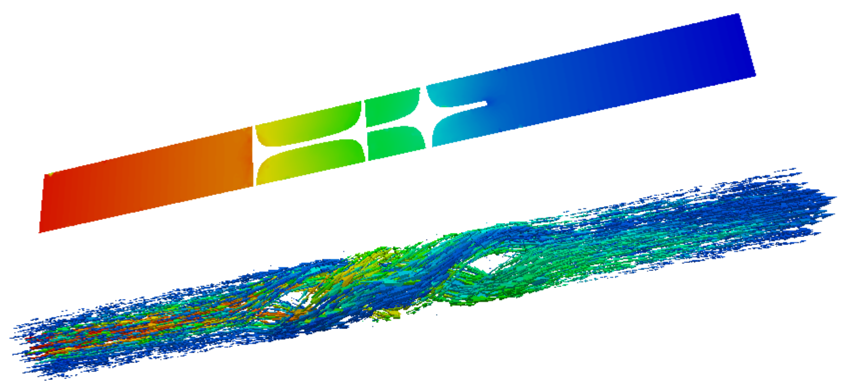
DEPARTMENT OF
EARTH SCIENCE AND ENGINEERING



FLUIDITY

- (CVD)FEM based Navier Stokes/Darcy flow solver framework
- Unstructured mesh adaptivity capability
- Multimaterial & multiphase formulations (and both together)

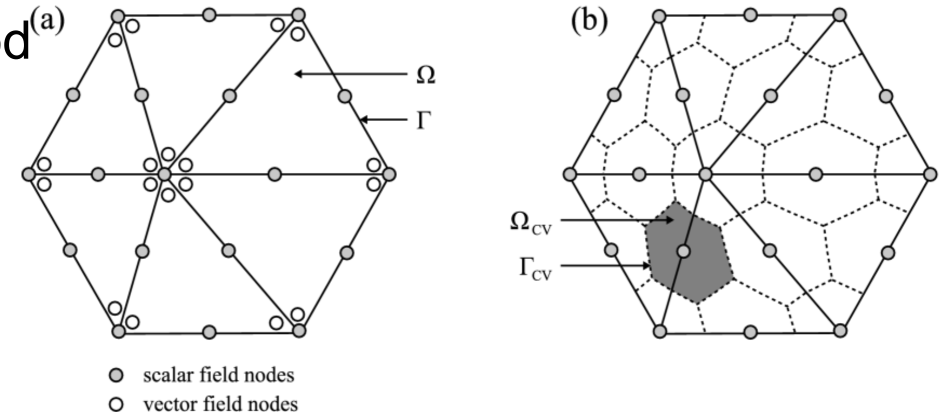


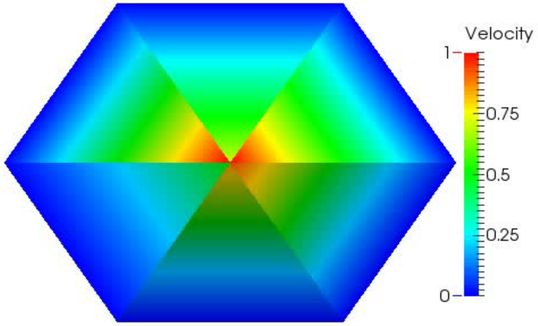


CVDFEM methods

Control Volume Finite Element Method

- Solve weak form PDEs
- piecewise smooth fns for unconserved, unbounded variables
- piecewise flat fns for conserved & bounded tracers

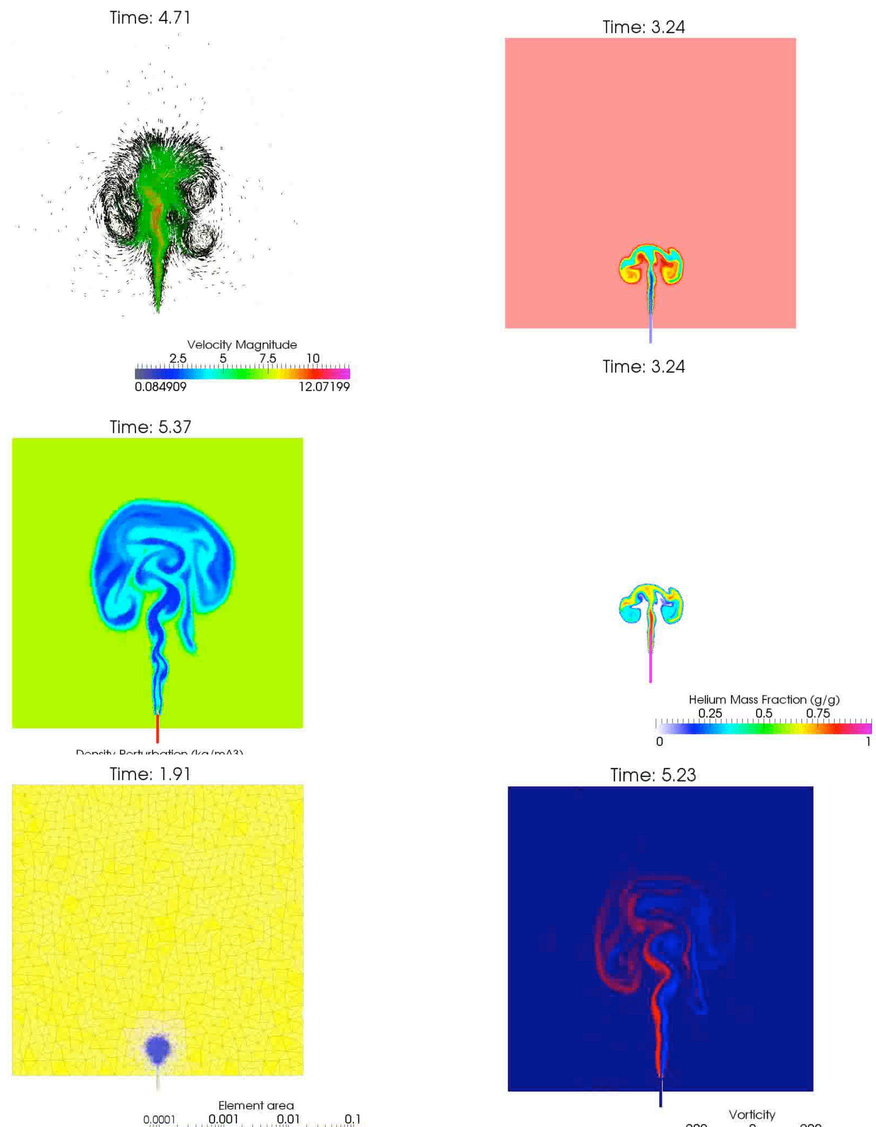




Discontinuous Galerkin Methods
Don't strongly enforce continuity of the finite element variables across finite element boundaries

Mesh adaptivity

- Hessian approach optimizes estimate of linear interpolation error in chosen variables for given number of DOFs



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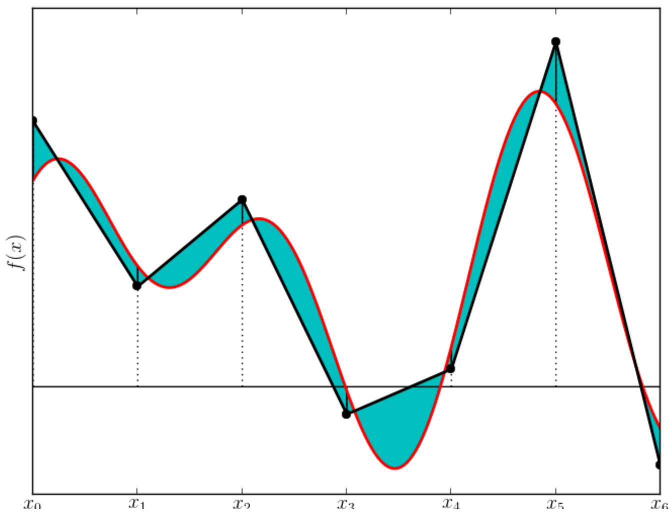
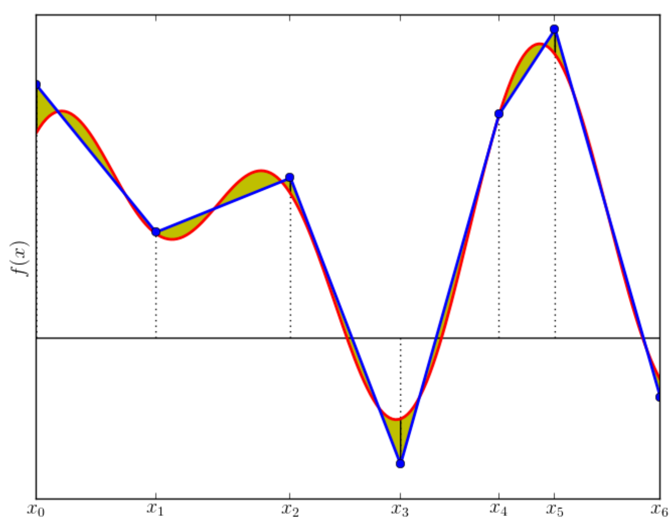
Mesh Adaptivity

$$F = \sum_{i \in \text{edges}} (\boldsymbol{v}_i^T \underline{\boldsymbol{M}}_i \boldsymbol{v}_i - 1)^2$$

$$[\underline{\boldsymbol{M}}]_{ij} = \det |H|^{-1/7} \frac{|H|_{ij}}{\epsilon}$$

$$[H]_{ij} = \frac{\partial^2 \alpha}{\partial x_i \partial x_j}$$





C.C. Pain, A.P. Umpleby,
C.R.E. de Oliveira & A.J.H. Goddard (2001)
Computer Methods in Applied
Mechanics and Engineering

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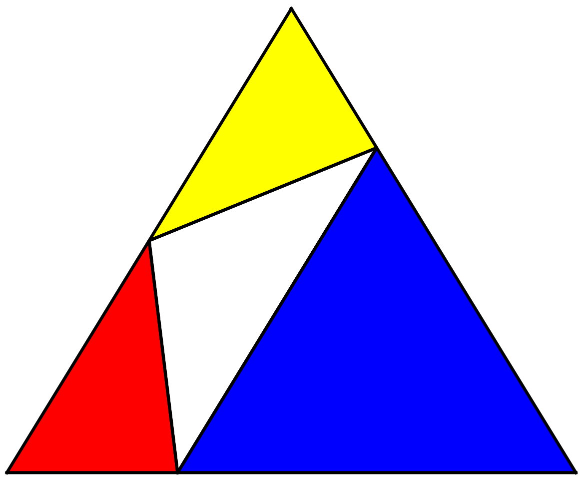
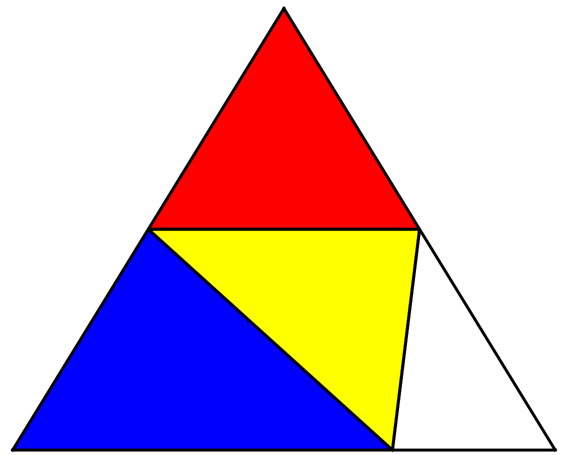
Supermeshing

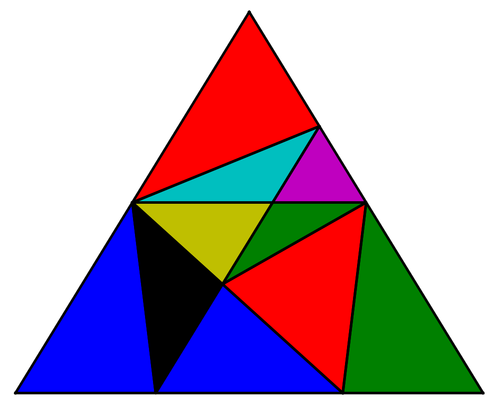
Old Mesh



New Mesh



Supermesh



$$\sigma : \left\{1, \ldots, \left|T^{(3)}\right|\right\} \rightarrow \left\{1, \ldots, \left|T^{(2)}\right|\right\}$$

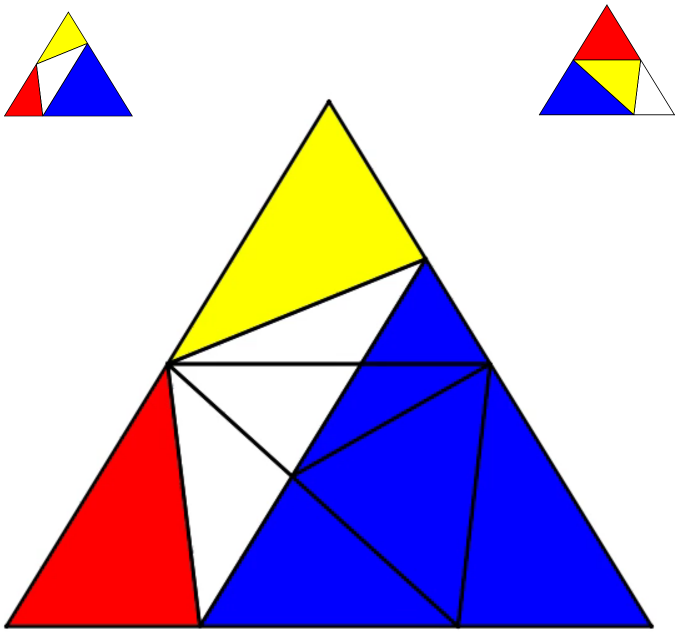
$$\pi : \left\{1, \ldots, \left|T^{(3)}\right|\right\} \rightarrow \left\{1, \ldots, \left|T^{(1)}\right|\right\}$$

$$\boldsymbol{x} \in T_k^{(3)} \Rightarrow \boldsymbol{x} \in T_{\pi(k)}^{(1)} \text{ and } \boldsymbol{x} \in T_{\sigma(k)}^{(2)}$$

Supermeshing

FE solutions/test functions piecewise smooth over mesh elements

Elements of supermesh: old variables and new test fns both smooth. No jumps.



Allows efficient conservative mesh to mesh interpolation via projection methods

P. E. Farrell & J. R. Maddison (2011) Computer Methods in Applied Mechanics and Engineering

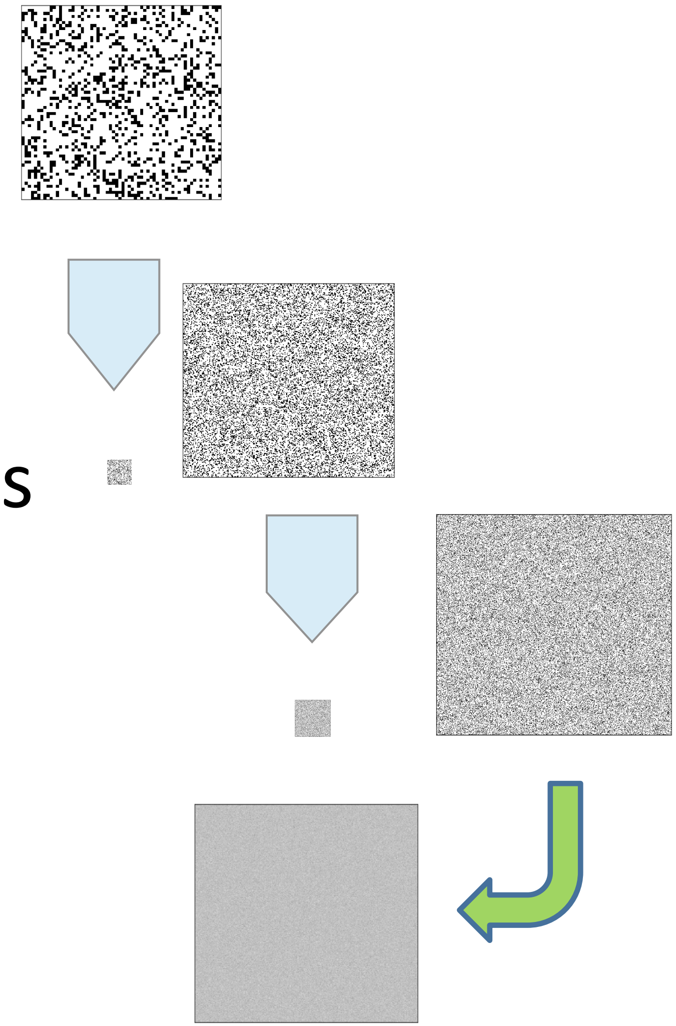
$$\sum_{k\in\{1,\ldots,|T_3|\}} \int_{\Omega_k} N_{\sigma_i(k)} N_{\sigma_j(k)} \hat{\alpha}_{\sigma_j(k)} dA = \sum_{k\in\{1,\ldots,|T_3|\}} \int_{\Omega_k} N_{\sigma_i(k)} N_{\pi_j(k)} \alpha_{\pi_j(k)} dA$$

Eulerian-Eulerian modeling

Numerical modeling implies existence minimum resolvable length scale

Droplets/bubbles/solid particles may be far below this scale.

Filter equations to homogenize into “two-fluid” model.



Eulerian-Eulerian modeling

General equations:

momentum:

$$\frac{\partial}{\partial t}(\rho_i\alpha_i\boldsymbol{u}_i) + \nabla\cdot(\rho_i\alpha_i\boldsymbol{u}_i\boldsymbol{u}_i) = -\alpha_i\nabla p_i - \rho_i\alpha_i g\hat{\boldsymbol{z}} + \boldsymbol{F}_i$$

continuity:

$$\frac{\partial}{\partial t}(\rho_i\alpha_i) + \nabla\cdot(\rho_i\alpha_i\boldsymbol{u}_i) = S_i$$

internal energy:

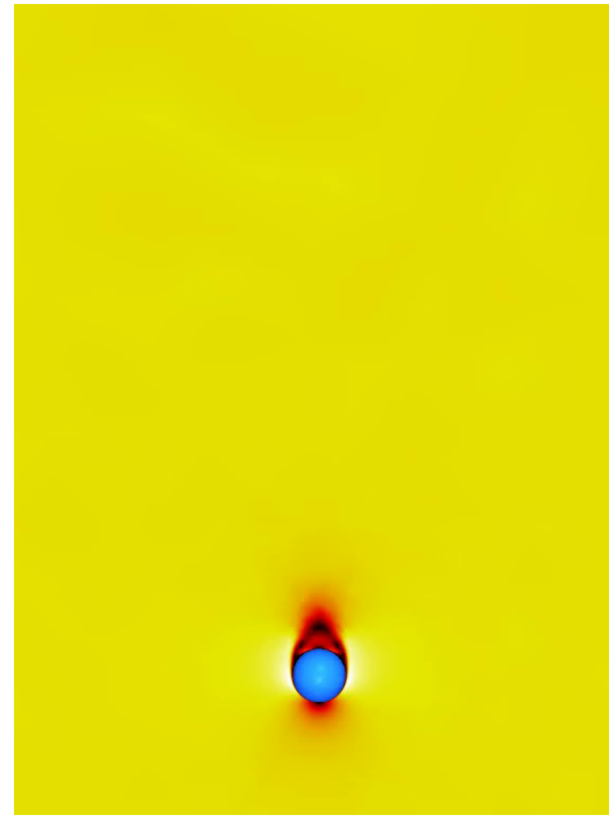
$$\frac{\partial}{\partial t}(\rho_i\alpha_i c_{p,i}T_i) + \nabla\cdot(\rho_i\alpha_i c_{p,i}T_i\boldsymbol{u}_i) + p_i\nabla\cdot\boldsymbol{\alpha_i u_i} = S_i$$

Interphase forces: drag

- Drag term: momentum exchange between phases

$$K_{\text{drag}} = \frac{\partial \boldsymbol{F}}{\partial \Delta \boldsymbol{u}} (\boldsymbol{u}_1 - \boldsymbol{u}_2)$$

- In limit of few, small particles: Stokes law
- Behaviour at higher concentrations less well determined.



$$v_{\text{terminal}} = \frac{2}{9} \frac{(\rho_d - \rho_c)}{\mu_c} g {d_p}^2$$

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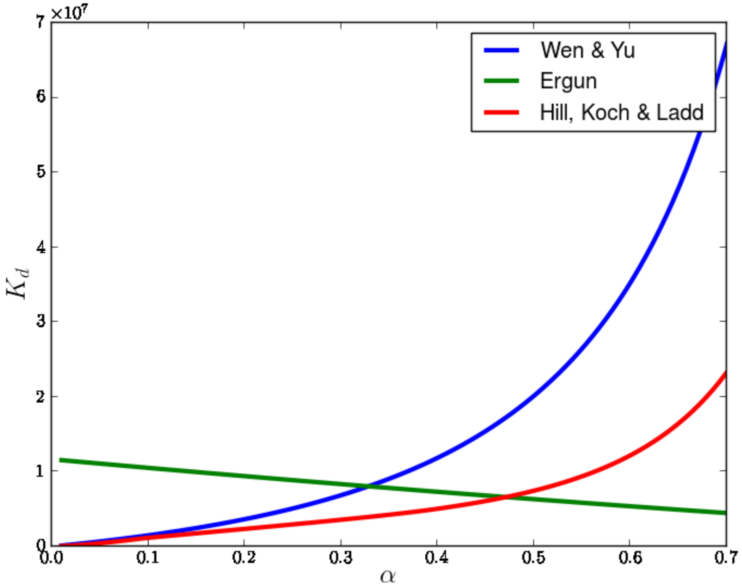
Interphase forces: drag

Many drag closure models exist:

- Ergun (1952)
- Wen & Yu (1966)
- Symlaml & O’Brien (1987)
- Gidaspow(1990)
- Hill Koch & Ladd (2006)

Variously from theory, numerical modelling or matching experiments.

Interphase forces: drag

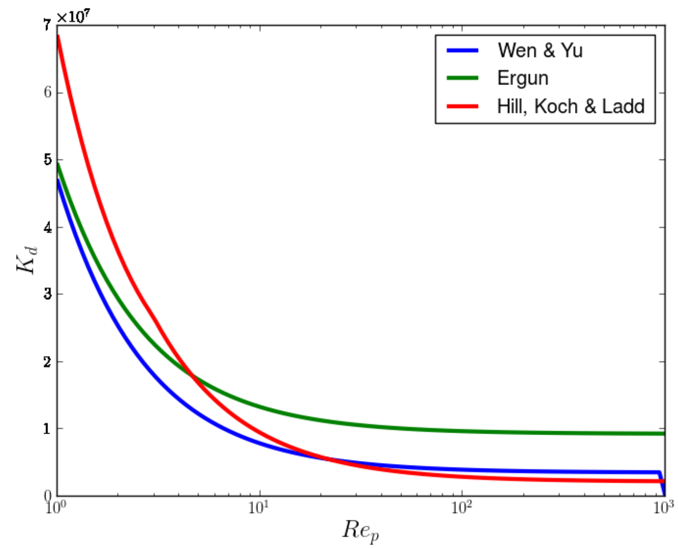


- Different closures appropriate for different systems
- Don't just implement a few, implement a framework

$$K_{\text{drag}} = \frac{\partial \boldsymbol{F}}{\partial \Delta \boldsymbol{u}} (\boldsymbol{u}_1 - \boldsymbol{u}_2)$$

$$\frac{\partial \boldsymbol{F}}{\partial \Delta \boldsymbol{u}} = f(\alpha_i, \boldsymbol{u}_i, \rho_i)$$

- Specify through scripting code



Interphase forces: drag

Example: Wen & Yu drag correlation

$$K_{\mathrm{drag}} = 3C_D/4\alpha_g\alpha_s\rho_g\frac{|\boldsymbol{u}_g - \boldsymbol{u}_s|}{d_s}\alpha_g^{-2.65}$$

$$C_D = \begin{cases} 0.44 & \mathrm{Re}_s > 1000, \\ \frac{24}{\mathrm{Re}_s}(1 + 0.15 * \mathrm{Re}_s^{0.687}) & \text{otherwise} \end{cases}$$

$$\mathrm{Re}_s = \frac{\rho_g\alpha_g|\boldsymbol{u}_g - \boldsymbol{u}_s|}{d_s}$$

Interphase forces: drag

Example: Wen & Yu drag correlation

```
def val(x,t,a_1,a_2,rho_1,rho_2,u_1,u_2):
   mu=1.0e-5
   d_s=150e-6
   Re_s=a_1*rho_1*abs(u_1-u_2)*d_s/mu
   if Re_s>1000:
       C_D=0.44
   else:
       C_D=24/Re_s*(1.0+0.15*(Re_s**0.687))
   return 3*C_D/4.0*a_1*a_2*rho_1*abs(u_1-u_2)/d_s*a_1**-2.65
```

Interphase forces: drag

Example: Wen & Yu drag correlation

```
\begin{algin}
        K_{\mbox{drag}}=3C_D/4\alpha_g\alpha_s\rho_g
                    \frac{|\bm{u}_g-\bm{u}_s |}{d_s}\alpha_g^{-2.65}\\
        C_D=\begin{cases}
                    0.44                                        & \mbox{Re}_s>1000,\\
                    \frac{24}{\mbox{Re}_s}
                          (1+0.15*\mbox{Re}_s^{0.687})   & \mbox{otherwise}
              \end{cases}\\
        \mbox{Re}_s=\frac{\rho_g\alpha_g |\bm{u}_g-\bm{u}_s |}{d_s}
\end{align}
```

Interphase forces: granular temperature

- Solid has
  - Max packing density
  - Quasi-elastic collisions
  - Additional kinetic energy in fluctuations
- Kinetic theory closure
- qv. Gidaspow(1994)

$$\boldsymbol{F}_{\text{solid}} = \nabla p_s + K_{\text{drag}} + \boldsymbol{F}_{\text{friction}}$$

$$\Theta_s = \frac{1}{3}\left\langle \left|\boldsymbol{u} - \boldsymbol{v}\right|^2 \right\rangle$$

$$p_s = \rho_s \alpha_s \left(1 + 2\left(1 + e\right)\alpha_s g_0\right)\Theta_s$$

$$g_0 = \left(1 - \left(\frac{\alpha_s}{\alpha_{s,\max}}\right)^{\frac{1}{3}}\right)^{-1}$$

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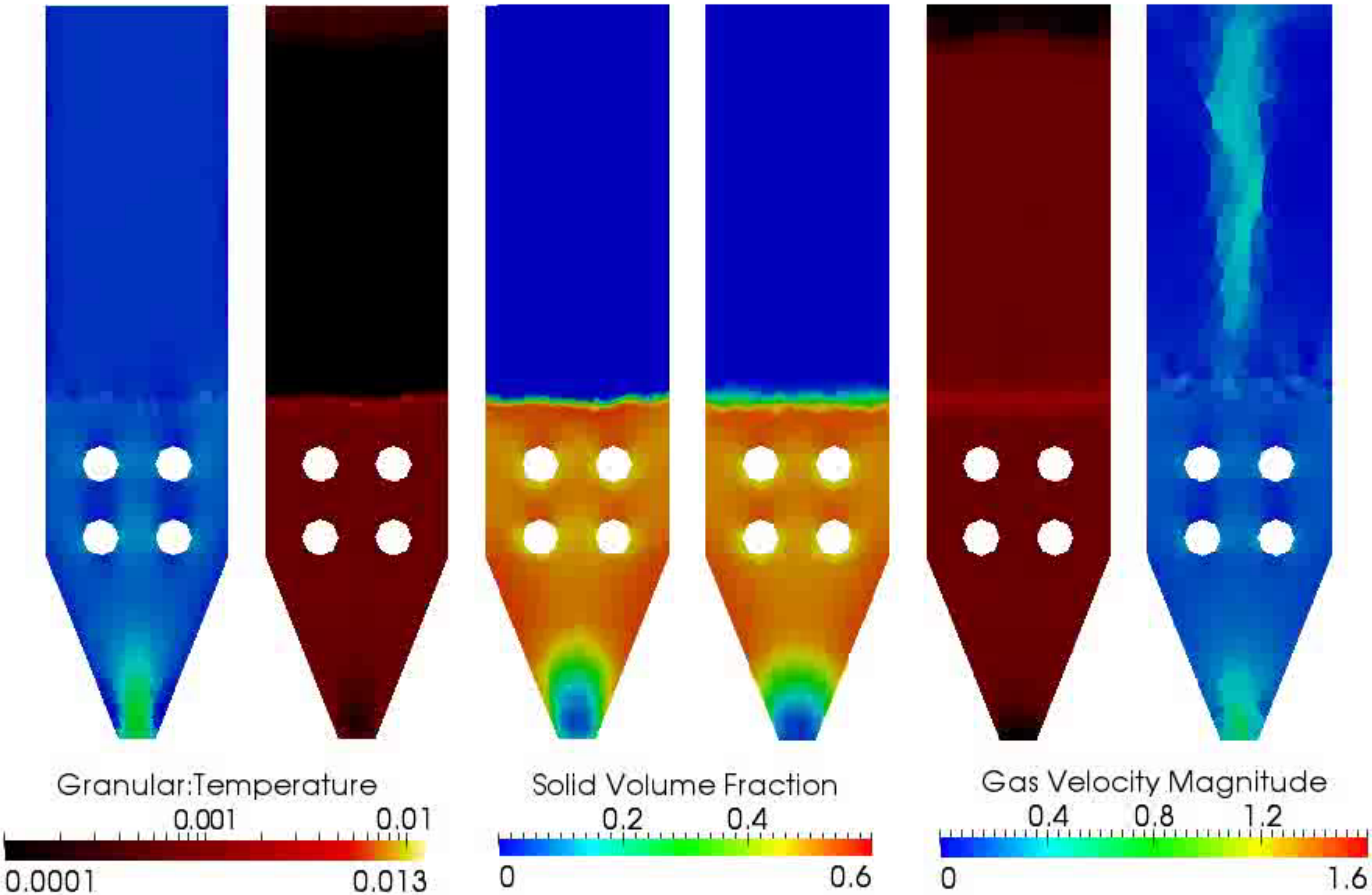
Interphase forces: granular temperature

$$\boldsymbol{F}_{\text{friction}} = \alpha_s \nabla \cdot \left(\mu_s \left(\nabla \boldsymbol{u}_s + \nabla \boldsymbol{u}_s \right) + \left(\lambda_s - \frac{2\mu_s}{3} \nabla \cdot \boldsymbol{u}_s \right) \boldsymbol{I} \right)$$

$$\mu_s = \frac{4}{5} \alpha_s d_s g_0 \left(1 + e\right) \left(\frac{\Theta_s}{\pi} \right)^{\frac{1}{2}}$$

$$\lambda_s = \frac{4}{3} \alpha_s d_s g_0 \left(1 + e\right) \left(\frac{\Theta_s}{\pi} \right)^{\frac{1}{2}}$$

$$\frac{\partial}{\partial t} \left(\rho_s \alpha_s \Theta \right) + \nabla \cdot \left(\rho_s \alpha_s \Theta \boldsymbol{u}_s \right) = \nabla \boldsymbol{u}_s \cdot \tau_s - \gamma \Theta - \frac{\partial \boldsymbol{F}}{\partial \Delta \boldsymbol{u}} \Theta$$



Granular:Temperature
0.001
0.01
0.0001
0.013
Solid Volume Fraction
0.2
0.4
0
0.6
Gas Velocity Magnitude
0.4
0.8
1.2
0
1.6

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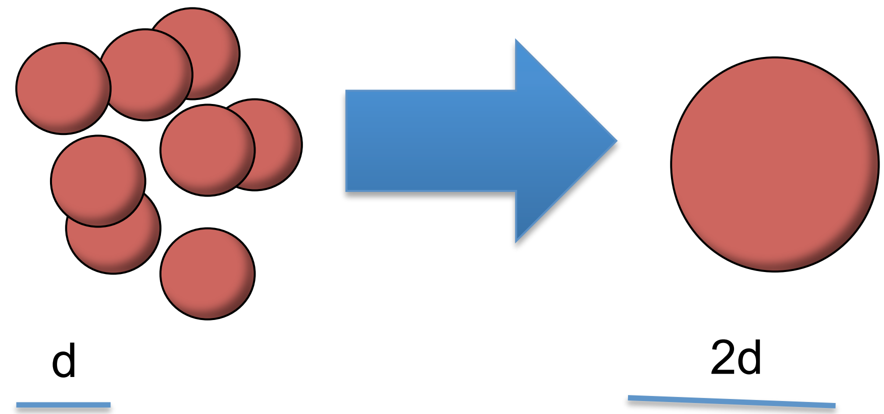
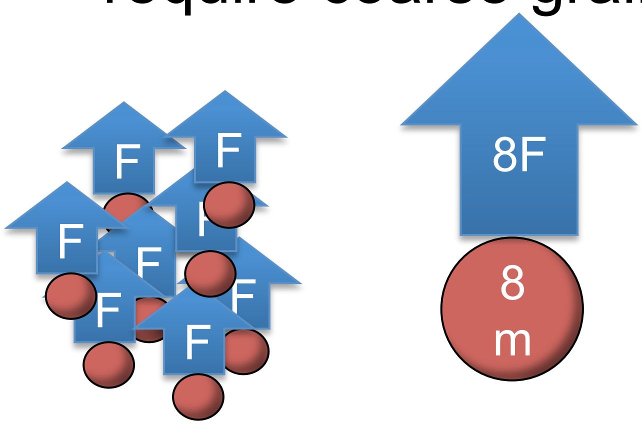
An Eulerian-Eulerian model of coarse graining

DEM modelling of large systems of particles can require coarse graining





Groups of small particles replaced with single large particle, retaining solid volume, COM and acceleration due to drag forces

How to use Eulerian-Eulerian models to understand the effects of coarse graining?

An Eulerian-Eulerian model of coarse graining

Use real particle diameter in drag correlation

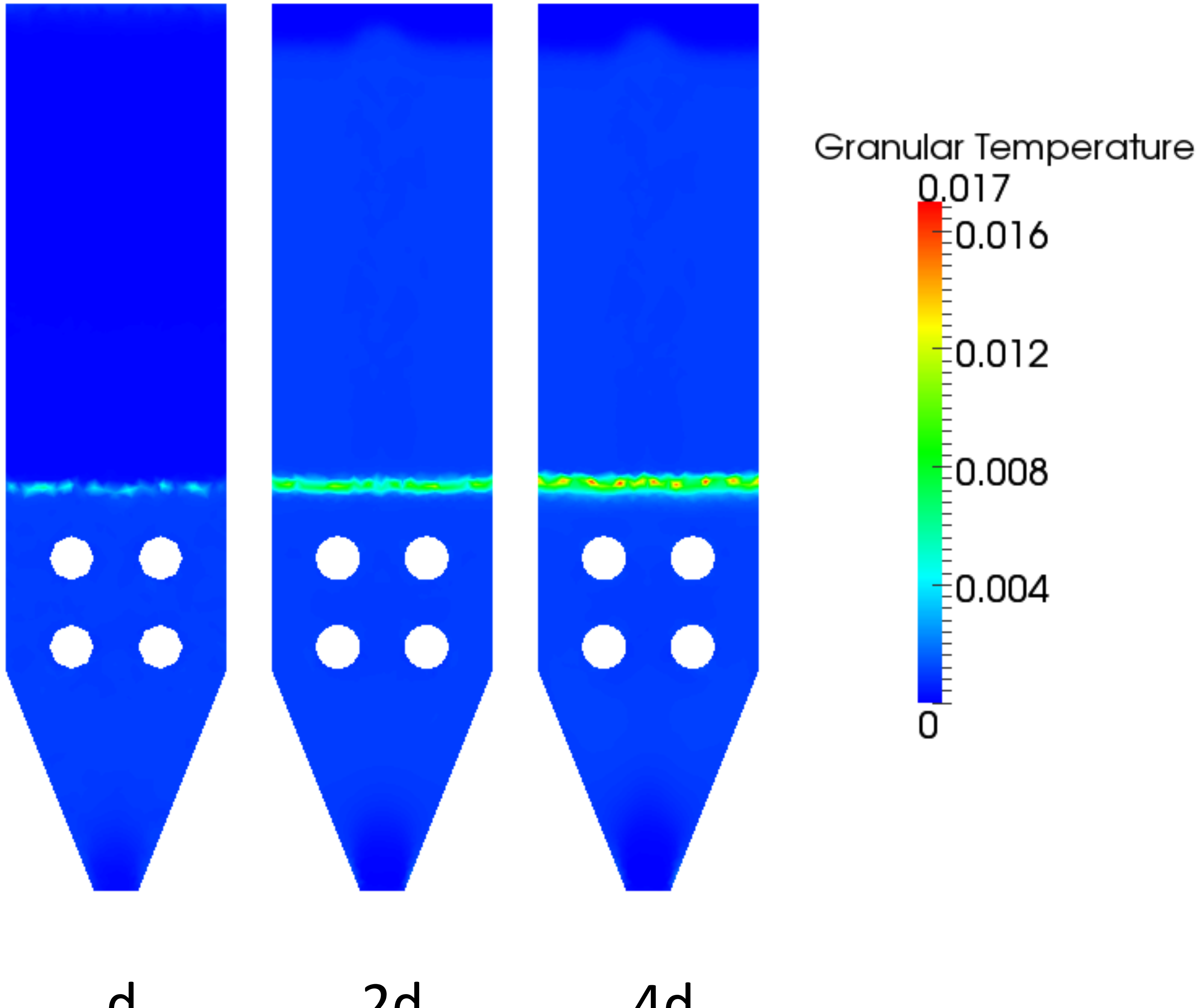
$$K_{\text{drag}} = 3C_D/4\alpha_g\alpha_s\rho_g\frac{|\boldsymbol{u}_g - \boldsymbol{u}_s|}{d_s}\alpha_g^{-2.65}$$

$$C_D = \begin{cases} 0.44 & \text{Re}_s > 1000, \\ \frac{24}{\text{Re}_s}(1 + 0.15 * \text{Re}_s^{0.687}) & \text{otherwise} \end{cases}$$

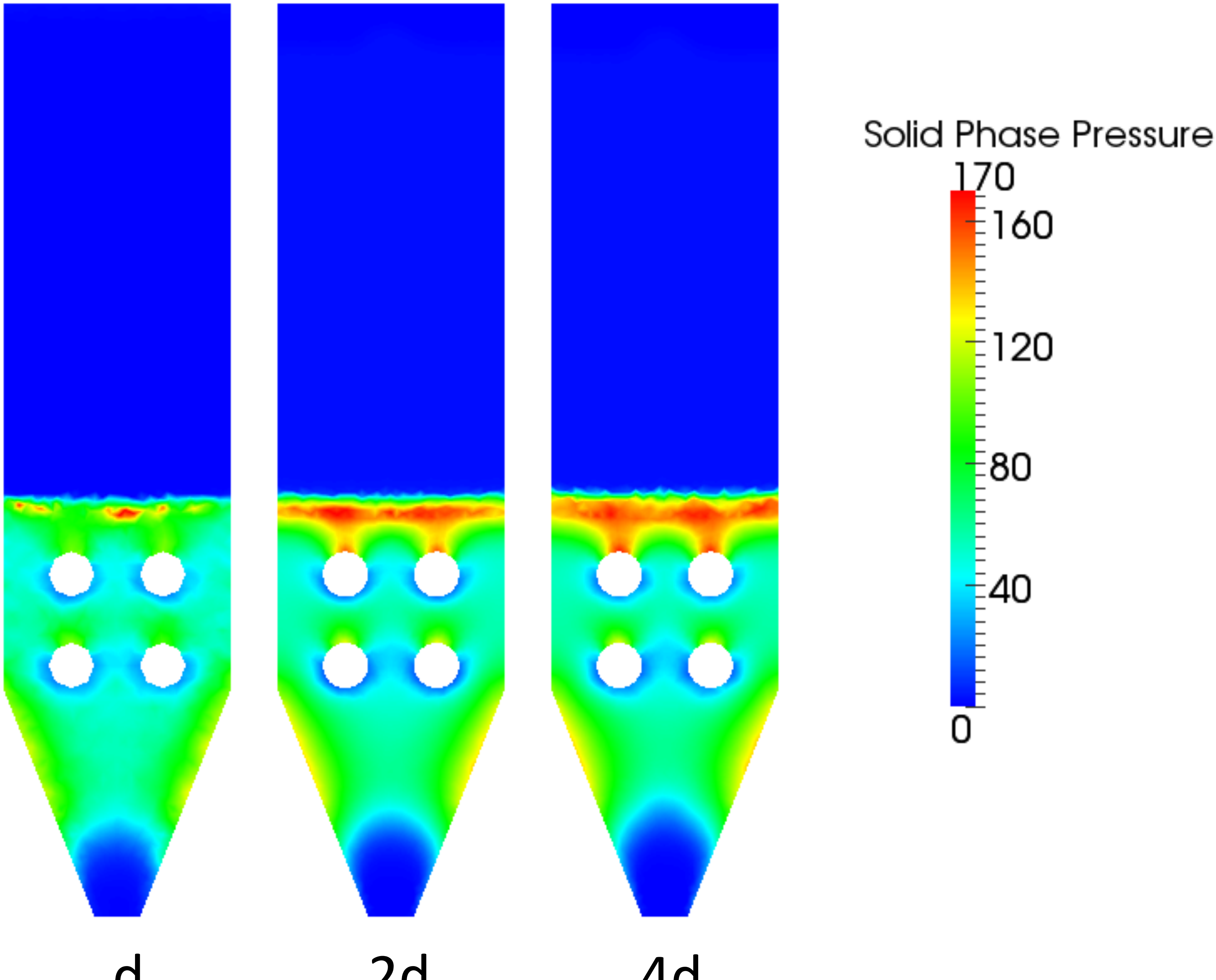
$$\text{Re}_s = \frac{\rho_g\alpha_g|\boldsymbol{u}_g - \boldsymbol{u}_s|}{d_s}$$

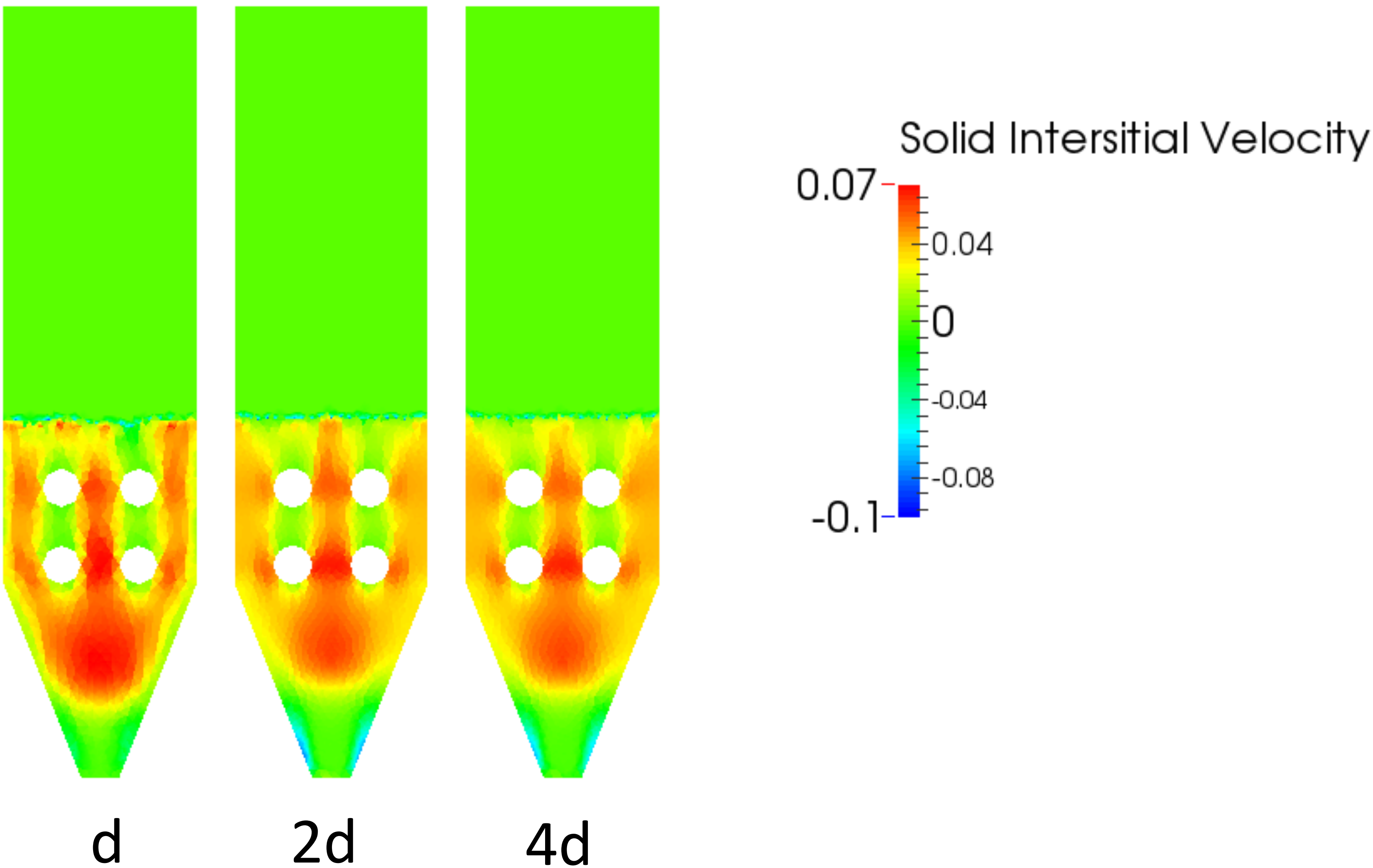
Use coarse particle diameter in collisional terms:

- Collisional Solids viscosity
- Collisional energy disipation



Granular Temperature
0.017
0.016
0.012
0.008
0.004
0
d
2d
4d


Solid Phase Pressure
170
160
120
80
40
0
d
2d
4d


Solid Intersitial Velocity
0.07
0.04
0
-0.04
-0.08
-0.1
d
2d
4d

Polydispersion

Multiple sizes/shapes of dispersed material

- Drag terms:
  - technology already there!
- Particle-particle interactions
- Size parameterization:
  - Binning approach

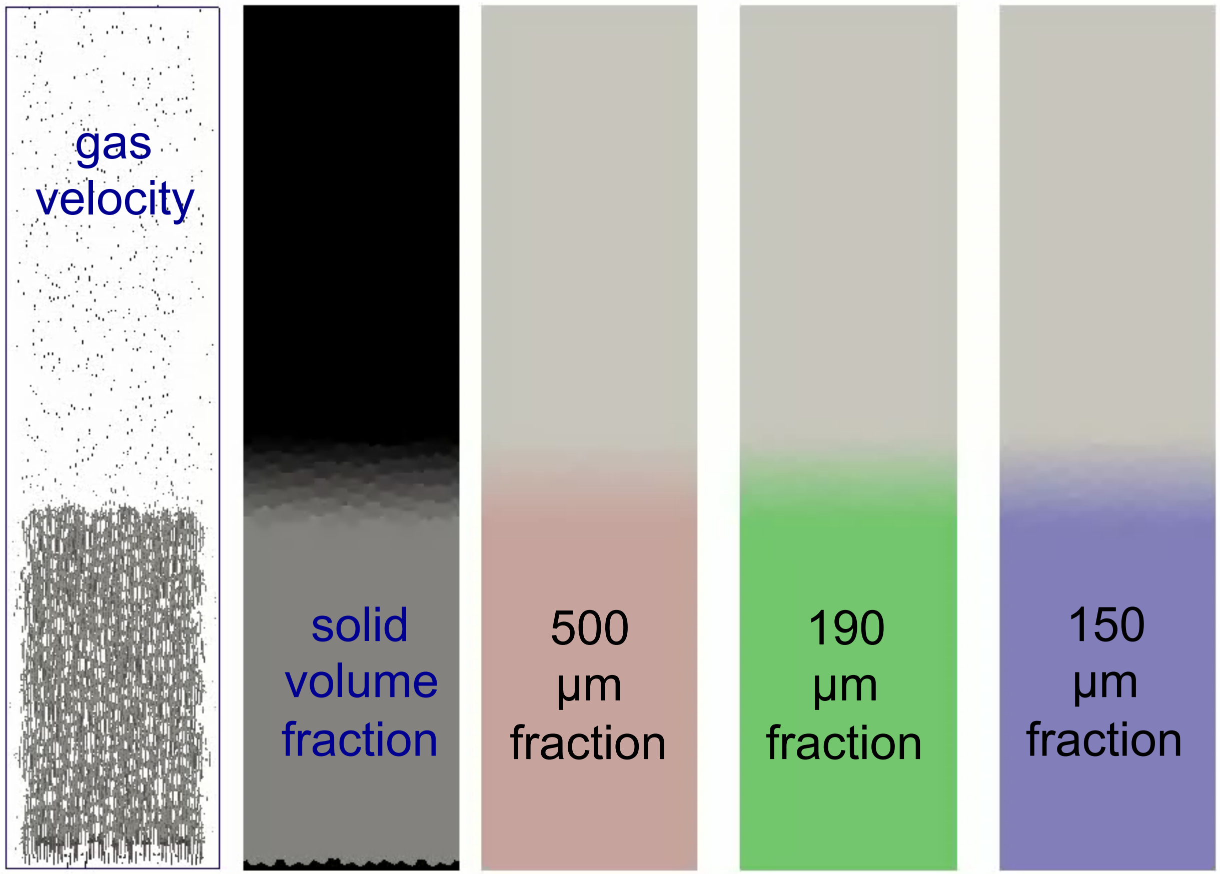
$$\frac{\partial}{\partial t}(\rho_i \alpha_i \boldsymbol{u}_i) + \nabla \cdot (\rho_i \alpha_i \boldsymbol{u}_i \boldsymbol{u}_i) = -\alpha_i \nabla p_i - \rho_i \alpha_i g \hat{\boldsymbol{z}} + \boldsymbol{F}_i$$

Polydispersion

Little consensus in literature:

- Single or multiple granular temperatures?
- Modification of maximum packing fraction for mixtures? Derive from binary mixture.
- Existing gas-solid drag term, or additional terms?
- Which solid-solid interactions to consider?
  - Solid-solid interphase drag? Symlaml 1987
  - Which phases contribute to solid viscosity?

Polydispersed Granular Mixtures: Single temperature



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Summary

- Presented a CVFEM Eulerian-Eulerian model for dense granular flows with mesh adaptive capability.
- Described extensions to modelling coarse-grained simulations using Discrete Element Methods
- Described extensions to modelling polydispersive granular flows

Thank you for your attention

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Work performed under EPSRC programme Grant EP/K003976/1